Tech-Driven Secular Stagnation: Cross-Country Evidence*

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Abstract

We investigate the role of technology on the global secular stagnation of average labor productivity (ALP). In the last decade, equipment-specific technological progress has been stagnated significantly across developed countries, reflected in the movement of the relative equipment prices. Using a multi-capital growth model, we quantify the effect of the equipment-specific technological stagnation on the ALP growth rates. Our analysis shows that the technological stagnation alone accounts for 71% of the declines in the US ALP and a bulk fraction of other developed countries. We discuss several possible causes behind the global equipment-specific technological stagnation.

Keywords: Economic Growth, Secular Stagnation, Technology, Relative Investment Price, Equipment

JEL Classification: O41, O47

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1 Introduction

Many developed countries are experiencing serious stagnation in economic activity, measured by average labor productivity (ALP). This fact is the most recognized about the United States (the US) and is often referred to as *secular stagnation*: the ALP growth of the US has been slowing down since around 2005 (Byrne et al. (2016) and Syverson (2017)). Spurred by the chronic stagnation of the US, a number of papers have been written to identify its causes. On the one hand, researchers have argued that the lack of demand, caused by the zero lower bound constraint on the nominal interest rate, is the cause of the economic stagnation (Summers (2015)). Other researchers have argued that the stagnation of technological progress is the cause of the recent economic stagnation (Gordon (2015)). These past works often focus their analysis on a specific country, mostly the US, while the stagnation is commonly observed across developed countries.

This paper seeks to find a *unified* explanation of why so many developed countries, which have different economic structures, regulations, and monetary policy, have simultaneously stagnated in the last decade. Our primary observation is that many developed countries are simultaneously experiencing equipment-specific technology stagnation, reflected in the movement of relative prices of equipment (Gordon (1990); Hulten (1992); Greenwood et al. (1997)). In contrast to the past trends, the relative prices of equipment have stopped falling across developed countries in the last decade. For example, the relative price of the US computer hardware had steadily declined by nearly 20% annually until 2005. But after 2005, the decline in the relative price has dramatically weakened. This fact about the relative price of the computer hardware holds for a number of developed countries, and similar phenomena also occur for other equipment. We refer to the global equipment-specific technological stagnation as *the technology stagnation.* This paper examines whether the technology stagnation alone can explain the ALP stagnation across developed countries.

To this end, we extend a standard growth model by incorporating multiple capital goods (e.g. computer hardware, structures, and so on). In the model, the firms which produce the different investment goods can have different total factor productivity (TFP) growth. Under the perfect competition assumption, the relative prices of the investment goods reflect the difference in the TFP levels. Exploiting this feature of the model, we estimate the sectoral TFP growth rates for each developed country. The estimated sectoral TFP of equipment have stagnated after 2005 for most of the developed countries, but those of non-equipment have not.

Before quantifying the effect of the technological stagnation on the ALP growth, we perform various validation exercises on our estimated model. We begin by verifying that the model can successfully reproduce the actual stagnation of ALP growth after 2005. For this exercise, we derive sufficient statistics of the changes in ALP growth rates along the balanced growth path (BGP), and confirm that the changes in the ALP growth predicted by the model decreases to the same extent as the actual declines of ALP growth. In addition, we provide two external validation exercises. Specifically, we examine whether the model can reproduce the changes in the growth rates of the real wages and the growth rates of the ratios of capital to labor across the countries. We find that the model predicts their changes across the countries accurately.

Having established the credibility of our model, we proceed by analyzing how much the technology stagnation alone has pushed down the ALP growth rate in each country. For this exercise, we again exploit the sufficient statistics used in our validation exercise. We find that the technology stagnation alone explains a bulk fraction of the observed declines in the ALP growth rates across developed countries. To provide a concrete number, the average ALP growth rate of the US declined by 1.17% points after 2005, and the model predicts that the technology stagnation lowers the US ALP growth by 0.83% points. Thus, more than 70% of the decline can be explained by this single factor. The technology stagnation is also severe for the UK: the technology stagnation accounts for 80% of the ALP stagnation. The ALP stagnation of other developed countries, not just the US and the UK, can be accounted well for by the technology stagnation. Given these findings, we conclude that we successfully identify a single factor that can explain the global ALP stagnation of all developed countries.

At this point, some readers might rightly wonder why the equipment-specific technology stagnation can explain a bulk fraction of the observed ALP although their shares in value added are small. It should be noted that the equipment goods are input for capital stocks. So, when technology stagnation happens to the equipment sectors, capital accumulation slows down, which further depresses the output. This indirect effect turns out to be powerful enough to explain the observed ALP stagnation.

For our findings to be valid, it is critically important that the relative equipment prices correctly reflect the equipment-specific technological changes. However, other factors unrelated to the equipment-specific technological changes may affect the relative prices of equipment. To shed light on this issue, we discuss three possibilities that might affect the relative investment prices. The first possibility is related to (mis)measurement. One may claim that mismeasurement in the official statistics caused the observed relative prices of equipment to stop falling, but the true relative prices have been falling. To argue against this possibility, we examine various past research addressing the mismeasurement issue (Byrne et al. (2016); Aghion et al. (2019); Brynjolfsson et al. (2019)). While these papers address different types of mismeasurement, they reach a very similar conclusion. The mismeasurement has always been an issue, but the amount of this mismeasurement is not increasing after 2005. So, we conclude that the mismeasurement is not sufficient for explaining the fact that the relative equipment prices stopped declining.

The second possibility is that trade may affect the relative prices of equipment. Since China rapidly expanded its export to developed countries after 2000, it is natural to imagine that this rise of China somehow affects the relative equipment prices. As an argument against this concern, we compare our estimate of the TFP with ones reported in the EU KLEMS dataset or comparable datasets. Since the productivity sequences reported in these datasets do not use the information on the relative investment prices directly, their estimates are less subject to the concern about trade. We show that their TFP estimates for the computer sector exhibit the same slowdown after around 2005.

The third possibility is related to the rise of market power. There is a growing concern that the US firms have acquired more market power (De Loecker et al. (2020)), and the rise of the relative prices of equipment may reflect the market power of equipment producers. This possibility is assumed away in our model since we have assumed perfect competition. Using the estimation method in Hall (2018), we examine whether the equipment sectors have expanded their market power. We provide industry-level evidence to prove otherwise. Moreover, De Loecker and Eeckhout (2021) examine whether the firms around the world have acquired more market power. According to their findings, the markup for some countries, for example Germany and Japan, have not increased in the last decade. So, this study is consistent with the view that the rise of market power did not cause the relative prices of equipment to stop falling across developed countries.[1]

We also provide an additional robustness exercise related to the output elasticities with respect to capital stocks. As we mentioned above, the output elasticities are critically important for our findings. Under our modeling assumption, the output elasticities correspond to the shares of capital goods in rental costs, which can be measured directly in principle. But, few countries have such estimates by category of capital goods so that we use the indirect method to estimate the output elasticities following Gourio and Rognlie (2020). Fortunately, the estimates of the rental cost for Japan and the US are provided by the Japan Industry Productivity Database and Bureau of Labor Statistics.[2] We redo our quantification analysis by using these shares. We find that the technology stagnation causes greater stagnation in ALP growth rates than our benchmark results. So, we conclude that our results are robust in this dimension too.

Having established the robustness, we then discuss which types of theories are consistent with our findings. As mentioned above, theories which explain the cause of stagnation based on a domestic factor are not consistent with our results. Instead, our findings support theories that feature global technological innovation. For example, a recent paper by Bloom et al. (2020) argues that innovation becomes increasingly difficult in various sectors, which is arguably true for developed

[1] We are currently working on estimating firm-level TFP using the method by De Loecker and Eeckhout (2021).

[2] The Japan Industry Productivity Database Database forms part of the EU KLEMS project and constructs a dataset comparable to the ones by the EU-KLEMS.

countries. Our findings reach a similar conclusion as theirs.

This paper is related to two lines of work. The first is the literature analyzing the recent slowdown of the economic activities in the US and other developed countries like Japan. Gordon (2015), Fernald (2015), Byrne et al. (2016), Cette et al. (2016), Fernald et al. (2017), and Ramey (2020) argue that the recent slowdown of the US economy stems from a problem of the supply side of the economy. Similarly, Hayashi and Prescott (2002) argues that the Japanese economy has slowed down after 1990 due to the slowdown of the TFP growth. Goodridge et al. (2018) study the UK economy and find that TFP has slowed down. On the other hand, Summers (2015), Summers (2016), and Eggertsson et al. (2019a) argue that the slowdown of the US economy are something to do with a lack of demand. Moreover, Caballero et al. (2008) and Illing et al. (2018) study the effects of banking (zombie-lending) and financial frictions on the slowdown, and Aoki et al. (2017) argue that that switching to bad equilibrium from good equilibrium may be a cause behind the slowdown of the Japanese economy. Anzoategui et al. (2019) connect the productivity slowdown of the US with the Great Recession. More recently, Liu et al. (2021) provide a theoretical model, explaining a lower growth caused by a low interest rate. This paper is different from these previous works since we provide a unified explanation of the global stagnation across developed countries.

Second, this paper is related to the literature which studies capital-embodied technological change. Hulten (1992) and Greenwood et al. (1997) argue that the technological changes have played a major role in the growth of the US after World War II. Karabarbounis and Neiman (2014) studies the implication of the technological change on the recent decline of the US labor share.[3] More recently, Gourio and Rognlie (2020) study the aggregation issue related to the aggregate investment price of the US. Following these papers, we identify the equipment-specific technological changes by studying the relative prices of equipment.

The rest of the paper is organized as follows. In Section 2, we introduce the dataset and present the empirical analysis. Section 3 builds the multi-capital model to interpret the empirical analysis and studies its properties. In Section 4, we use the model to estimate productivity and provide the validation exercises. In Section 5, we use the model to quantitatively analyze the extent to which the technological stagnation of the equipment sector reduces the ALP growth rate across the countries. In Section 6, we provide additional evidences on our findings. In Section 7, we discuss what caused the slowdown of productivity and Section 8 concludes.

[3]There is an ongoing discussion on whether or not and why the labor shares across developed countries have decreased in the first place. See Bridgman (2018), Koh et al. (2020), and Gutiérrez and Piton (2020) and the references therein.

2 Empirical Facts

In this section, we provide a brief description about our datasets and then present empirical facts about developed countries. In particular, we focus on the growth rate of ALP and the relative prices of investment goods.

2.1 Datasets

We mainly use the EU KLEMS Release 2019 and comparable datasets (the KLEMS dataset for short), which provides a database on measures of economic growth, productivity, labor input, and capital input at the industry level for all European Union member states, Japan, and the US. [4] The KLEMS dataset provides the real capital stock sequences, the associated deflators by investment goods and labor service index for each country.[5] The KLEMS dataset apply quality adjustments to labor input in a standardized manner, which allows us to make international comparisons more easily. We supplement our dataset by adding various nominal sequences and consumption deflators from national accounts.

The main focus of our analysis is on developed countries that are considered to be at a similar technological level. For this reason, we restrict our analysis to countries which joined the OECD by 1995 and have the KLEMS dataset.[6] There are 13 countries which satisfy these criteria. Among these countries, we pay special attention to Germany, Japan, the UK, and the US, which are the four biggest economy.

Following Greenwood et al. (1997), we exclude the durable consumption durable from the consumption deflators by applying Tornqvist approximation method. Also we only consider investment and capital goods except the residential investment. Additional exceptional handlings are discussed in detail in Appendix A.

2.2 Empirical Facts

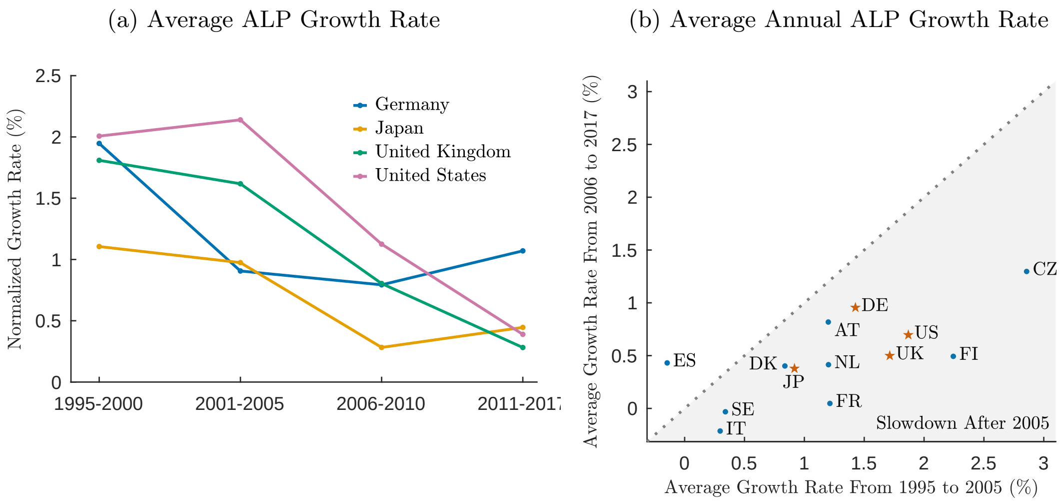
Now we proceed by documenting the empirical facts among the developed countries.

[4]The datasets can be downloaded from this website, https://euklems.eu/. For the US, we use the Integrated BLS-BEA Industry-level Production Account, which can be downloaded from this URL, https://www.bls.gov/mfp/mprdload.htm. For Japan, we use the Japan Industrial Productivity Database 2021 (JIP). These datasets corresponds to Japanese and the US counterpart of the EU-KLEMS dataset respectively. JIP and BEA-BLS construct their datasets in a methodology consistent with that of the EU-KLEMS.

[5]There are ten investment goods in the KLEMS dataset. They are: computing equipment; communications equipment; computer software and databases; transport equipment; other machinery and equipment; total non-residential buildings and structures (structures); residential structures; cultivated assets; research and development; and other IPP assets. See Gourio and Rognlie (2020) which study more disaggregated series for the US while such detailed information can be obtained for few countries.

[6]Canada has its KLEMS dataset, but it only contains data up to 2008. For this reason, we exclude Canada from our analysis.

Figure 1: Cross-Country Evidence



Notes: The annual growth rates of ALP are calculated by subtracting the growth rate of labor input from the growth rate of real GDP. Then we take the average growth rates of ALP every five years. The annual growth rate of the inverse of the relative price of computing equipment is calculated by subtracting the growth rate of the consumption deflator from the growth rate of the computing equipment. See Section 2.1 for a detail description of these variables.

Fact 1 : ALP Growth Rates Have Declined Globally

We begin by examining the evolution of the growth rates of ALP for Germany, Japan, the UK, and the US depicted by Figure 1a. The ALP growth rate of the US before 2005 had been high even by historical standards. This period of rapid ALP growth is often associated with the rise of information technology and its diffusion. However, since around 2005, the US ALP growth rate have slowed down gradually. While it is very difficult to pinpoint when ALP of the United State began to stagnate, Fernald (2015) argue that the slowdown precedes to the Great Recession.

The other three countries have experienced similar ALP slowdown since around 2005 to the US. It is worthwhile to mention that the Japanese slowdown of ALP came *after* the banking crisis of 1997. Also, Japan has faced another economic challenge since 1999, which is the zero lower bound constraint on the nominal interest rate. These Japanese experiences supplement Fernald (2015)'s argument. That is, the occurrence of a financial crisis, banking crisis, or the zero lower bound constraint is not necessarily related to slowdown in ALP.

Figure 1 shows that other developed countries have experienced the same slowdown ALP growth. The shaded area in Figure 1 indicates the areas where the average ALP growth rate slowed down from 2006. For countries other than Spain, the growth rate of ALP has slowed down while the magnitude of the slowdown varies from country to country.

These figures help us to understand the causes of this global stagnation. The following four points are worth special attention. First, the global stagnation of ALP is not necessarily related

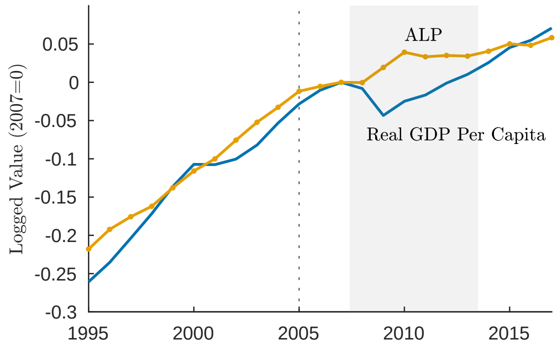


Figure 2: ALP and Real GDP Per Capita of the US

to economic crises. Fernald (2015) argues that the slowdown in the growth rate of ALP in the US started before the Great Recession occurred and predates the so-called housing market bubble. The experience of Japan provides a supplement argument for Fernald (2015). Japan has faced a number of severe economic challenges in the 1990s. Japan experienced the crash of the stock market in 1990, the non-performing loan problem since then, and the Asian currency crisis and the financial crisis in 1997. However, as shown in Figure 1a, it was much later than 2005, when the non-performing loan problem was resolved, that the growth rate for Japan slowed. So, the connection between the slowdown and financial crises is ambiguous.

By comparing ALP and real GDP per capita in the United States, we can see why Fernald (2015) and Syverson (2017) argued that the stagnation started in 2005, not 2007 when the financial crisis occurred.Figure 2 shows that US ALP did not begin to stagnate after the financial crisis, and appeared to start stagnated before the financial crisis. On the other hand, real GDP per capita began to stagnate sharply after the financial crisis. Since we study the stagnation of ALP, we use 2005 as the benchmark year when the stagnation started, following the past studies.

Second, the role played by the zero interest rate constraint is also limited, which is evident from the monetary policy of the US and Japan. The stagnation in the US began in 2005, but the zero interest rate policy was not implemented until 2009. On the other hand, in Japan, the zero interest rate policy has been in place since 1999, but the stagnation of Japanese economy had not accelerated until 2005.

The third point is that the slowdown of the ALP growth rates is not simply coming from the composition effect of labor. Japan and other developed countries have recently begun to experience rapid aging of the population. So, it sounds plausible that the aging of the population has led to a decline in labor productivity as workers flow into the medical and elderly care sectors, which are often regulated and may considered to have low productivity growth. By using the KLEMS labor service index, we can reduce the impact of such composition effects driven by the demographic changes on labor input. To put it simply, the growth rate of the KLEMS labor index is the sum of the growth rate of hours worked in each industry, with nominal wages as weights. Therefore, the contribution of working hours growth in industries with low wages to the labor index is moderated. This property ensures that even if workers flow to industries with more regulations, their contribution to the labor index will be discounted.

Finally, while it is clear that the slowdown of the ALP growth rates a global phenomenon, the timing of when the slowdown began differs slightly from country to country. Following Fernald (2015) and convention used in Syverson (2017), we take year 2005 as our benchmark year. That is, we presume that the economic slowdown started from 2005 globally. Since this benchmark year works well for the US and not so for the other countries, we provide robustness exercises by taking year 2007 as our benchmark year.

Fact 2 : Capital-embodied Technological Progresses of Computer Equipment Have Stagnated

We now explore a possible cause for the global stagnation of ALP. Motivated by the previous studies on growth (e.g. Greenwood et al. (1997)), we focus on capita-embodied technological progresses. the movements in the growth rate of the inverse of the relative prices of investment goods. Since the real investment is computed by dividing nominal investment by the relevant investment deflator, the inverse of the relative investment price is a proxy for technology improvement which is embodied in capital. [7]

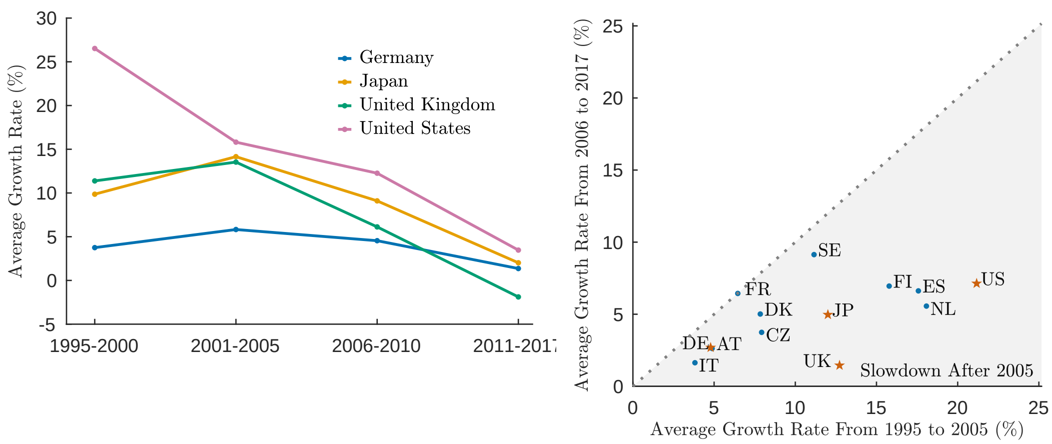
As shown in Figure 3a, Germany, Japan, the UK, and the US have experienced slowdown of technology improvement of computer. We begin by confirming that the same pattern is observed across the developed countries. Figure 3b shows the average growth rate of the inverse of the relative PC prices before and after 2005. The gray area represents the area where the growth rate of the relative computer price has stagnated since 2005. Notice that all the dots are located in the gray area, which implies the possibility that all the countries have experienced slowdown of computer technology.

The focus of our analysis is the slowdown of the technological improvement shown in Figure 3b. But, a reader might be wondering why the *level* of the growth rate of the relative computer price differs so much across the countries. To explore the cause behind this level difference, we construct an average relative imported personal computer (PC) price downloaded from the ComTrade dataset. Figure **??** depicts the average growth rates of the inverse of the relative imported PC prices for Germany, Japan, the UK, and the US. Figure **??** shows that the these growth rates have been stable. Also the level difference of the growth rates is much smaller than one of the quality-adjusted relative

[7] We introduce a micro-foundation for this proxy variable. As Greenwood et al. (1997), the model predicts that the proxy corresponds to the TFP level of computing equipment relative to the consumption good producer.

Figure 3: Technological Progress Specific to Computer Hardware

(a) Selected Countries (b) All Countries



computing equipment prices (See Figure 3a.) So, we conclude the quality-adjustments done by the statistical agencies play a major role for inducing the level differences.[8]

Our interpretation, which is consistent with the model below, is that the relative price of certain good reflects the relative technology level. Of course, the consumption and investment deflators contain the information about traded goods. So, the decline of a relative price might reflect the fact that Japan successfully buy cheaper durable and ICT goods from other country like China, not the stagnation of its relative technology level.[9] To resolve this issue and assess the robustness of our interpretation, we compare our measures of the technology stagnation of ICT goods with the TFP estimated by the KLEMS.[10] The estimates of TFP growth by the KLEMS is not recovered from the change in the relative prices of the final consumption or investment expenditure. Instead, the KLMES begins by measuring all the sectoral inputs and outputs.[11] By assuming that all the markets are competitive and the production function satisfies the constant-returns-to-scale, the sectoral growth rates of TFP are obtained by the growth accounting. So, the estimates by the KLEMS are not subject to the critique that our method faces.

We compare our measures of technology improvement of ICT goods sector with the sectoral TFP in the KLEMS of which the closest sector to our ICT goods sector. We choose the sector

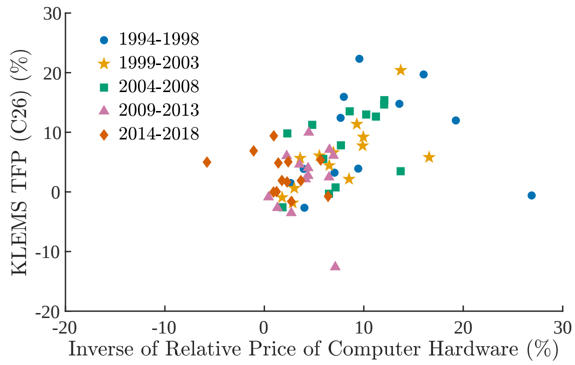
[8] One reason behind this level difference is that many countries use the hedonic method to adjust the quality of PC, but each country does not run the exactly same hedonic regressions.

[9] Another possibility is that the relative prices of these goods declined because the yen has depreciated, not the technology stagnation (**?**). Note that the depreciation of the yen from 2011 was temporary. From 2014, the nominal and the real effective exchange rates stopped depreciating. Still, the relative price did not start falling again. So, it is impossible to explain the *growth* rates of the relative prices of the durable and ICT goods based on the depreciation of the yen. See Appendix **??** for a detailed discussion on this issue.

[10] The Japanese version of the KLMES is called Japan Industrial Productivity Database (JIP). The latest version can be retrieved from the following link, https://www.rieti.go.jp/en/database/JIP2021/index.html.

[11] Of course, deflators are used for estimating real inputs and outputs. So these deflators play a role implicitly.

Figure 4: Comparison



Notes: Each point represents the average growth rate over a given period for a given country. Since we use the KLEMS data, our sample includes Austria, Belgium, Czech Republic, Denmark, Finland, France, Italy, Japan, Netherlands, Sweden, the United Kingdom, and the United States. For Japan, we have more disaggregated sectoral TFP estimates. So, we choose the sector called "Electronic data processing machines, digital and analog computer equipment and accessories."

called Computer, electronic and optical products (C26) from the KLEMS. In Figure 4, we plot the average growth rate of our measures of the ICT technology improvement in the horizontal axis and ones by the KLEMS in the vertical axis over various periods. Note that both measures show that the technology improvement of the ICT sector exhibits stagnation. These two measures positively correlate with each other and the naive regression tells that the slope is significant and positive. So, our interpretation that the technology slowdown of the ICT goods has happened globally is robust.

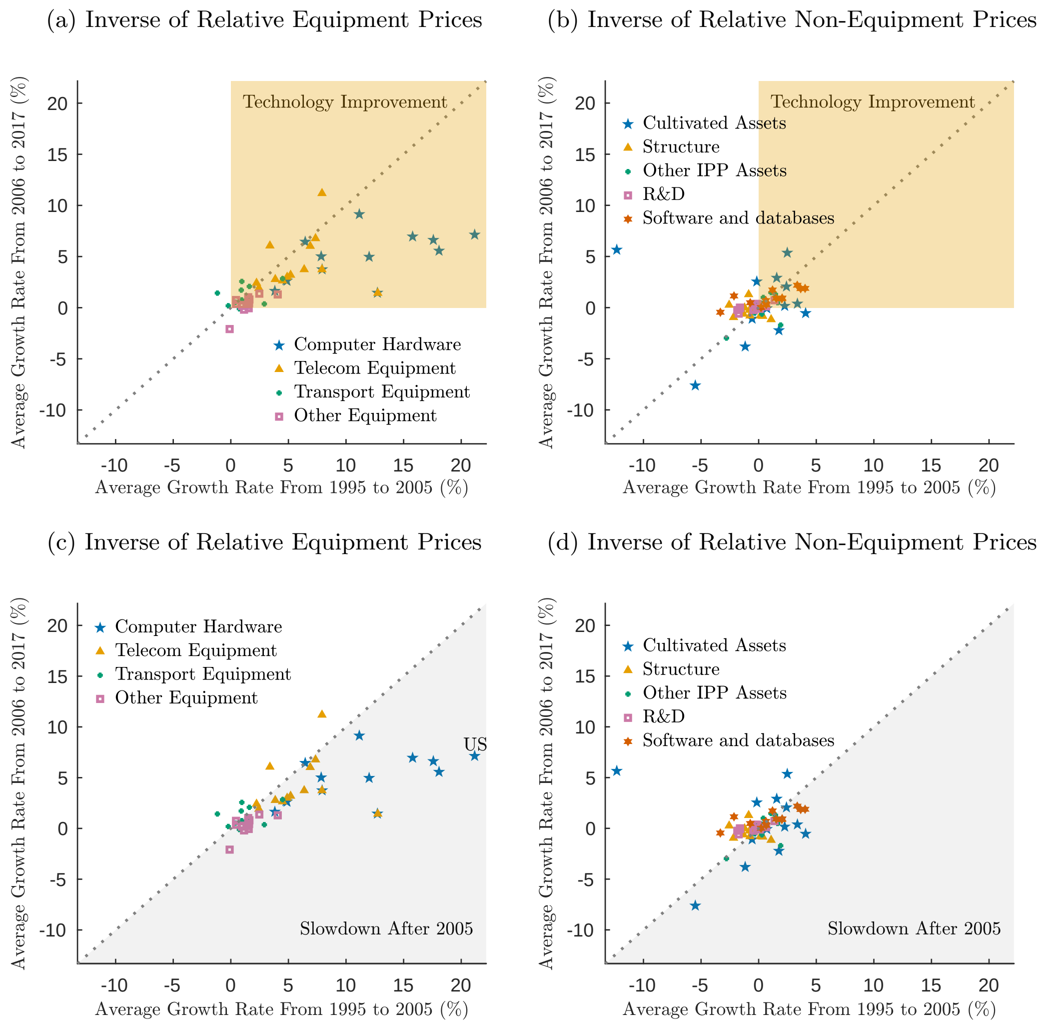
Fact 3 Heterogeneity of Capital-embodied Technological Progresses Across Asset Classes

We now explore the capital-embodied technological progresses of the other investment goods. Figure 5a depicts the growth rate of the inverse of the relative prices of various equipment. The areas where the technology of the equipment has improved both until and after 2005 are shaded orange. Figure 5d displays the same growth rates for the non-equipment investment goods. Figure 5a and 5d clearly show that the equipment investment goods have experienced stable technological improvement, but the other non-equipment investment goods have not.

Figure 5c and 5d are the same as Figure 5a and 5b respectively, except that the different parts are shaded. The gray areas represents the area where the technological improvement slows down after 2005. Again, the equipment goods have a consistent pattern: the technology improvement slowed down. The non-equipment goods do not have the consistent pattern.

Following Greenwood et al. (1997), we attribute the declines in the relative prices of the equipment goods to technological stagnation. The following sections build a growth model based on this

Figure 5: Heterogeneity of Capital-embodied Technological Progresses



Notes: The horizontal axis of this figure shows the average growth rate of the relative price by 2005 for each country. The vertical axis of this figure shows the same average growth rate after 2005.

interpretation, and analyze the stagnation of technological progress quantitatively.[12]

3 Model

In this section, we introduce our simplest-possible growth model in order to analyze the ALP and relative investment prices for each asset jointly. The model presented in this section extends the model of Greenwood et al. (1997) and Whelan (2003) by introducing many investment goods, and is closest to the one of Gourio and Rognlie (2020). The model is deliberately simple so that we skip some of the derivations. We explain how we generalize the growth accounting exercise in Section 3.2 and study the properties of the model in Section 3.3. We conclude this section with a discussion

[12]Readers may wonder whether this stagnation in the relative prices stems from some kind of measurement problem. This mismeasurement hypothesis will be discussed in detail in Section 6.1.

of the assumptions of the model.

3.1 Model Description

The time is discrete and indexed by t, and the economy is deterministic. There are two types of agents in this economy: households and firms. We explain their behaviors separately below.

Household

There is a representative household whose utility function is given by

$$U = \sum_{t=0}^{\infty} \beta^t \ln C_t, \tag{1}$$

where C_t is consumption at date t and β is a discount factor. The household inelastically supplies its effective labor, L_t and obtains the labor income $w_t L_t$. There are A classes of capital assets, and let $\mathcal{I}$ denote the set of the capital assets, $\mathcal{I} = \{1, 2, \cdots, A\}$. Let $\mathcal{E} \subset \mathcal{I}$ denote the set of the equipment goods. The capital stock of asset class a installed at date t is denoted by $K_{a,t}$. The household is the owner of the capital stocks and rents them to the final good producers. The rental rate of capital asset a is denoted by $r_{a,t}$. The household buys investment goods of asset class a at a price of $p_{a,t}$ each. The resulting flow budget constraint at date t is given by

$$p_{C,t} C_t + \sum_{i \in \mathcal{I}} p_{i,t} K_{i,t+1} \leq w_t L_t + \sum_{i \in \mathcal{I}} \left(r_{i,t} + (1 - \delta_i) \, p_{i,t} \right) K_{i,t}. \tag{2}$$

where $p_{C,t}$ is the price of the consumption good at date t and δ_i is the depreciation rate of capital good $i \in \mathcal{I}$. We take the consumption good as our numeraire good, $p_{C,t} = 1$. The household maximizes its utility (1) subject to the flow budget constraints (2).

Firms

Let $\mathcal{N}$ denote the union of the set of the consumption good, C, and the asset classes $\mathcal{I}$. We call an element of $\mathcal{N}$ a sector. We assume that there is a representative firm in each sector $n \in \mathcal{N}$, which have a constant-returns-to-scale (CRS) Cobb-Douglass production with the same factor shares:

$$Y_{n,t} = A_{n,t} \left(\prod_{i \in \mathcal{I}} K_{i,n,t}^{\theta_a} \right)^{\alpha} L_{n,t}^{1-\alpha}, \tag{3}$$

where $A_{n,t}$ is sector n specific technology level, $K_{i,n,t}$ is the capital stock of asset class a rented to the firm in sector n at date t, and $L_{s,t}$ is the labor input. The capital service sequence in sector n corresponds to $\prod_{i \in \mathcal{I}} K_{i,n,t}^{\theta_i}$, which is a geometric average of real capital stocks.

Firm n buys factor inputs, $\{K_{i,n,t}\}_{i\in\mathcal{I}}$ and $L_{n,t}$, in respective competitive factor markets, and sells its product in a competitive final good market. They maximize their static profits at each date t given the factor prices, $\left((r_{i,t})_{i\in\mathcal{I}}, w_t\right)$.

Real GDP

Most macroeconomic models assume that there is a single good in the economy. This simplifying assumption allows us to define the real GDP of the economy straightforwardly. When a model has more than one goods, then there are in principle many ways to define the real GDP. In this paper, we only define the real GDP growth rate, not real GDP level. To define the real GDP growth, it is convenient to use the following notation. Let $g_{X_t} = \ln(X_t/X_{t-1})$ denote the growth rate of the variable X at date t.

The real GDP growth rate, $g_{V_t^*}$, is defined as follows:

$$g_{V_t^*} = \sum_{n\in\mathcal{N}} s_{n,t-1} g_{Y_{n,t}}, \tag{4}$$

where $s_{n,t-1}$ is the share of sector n in value-added at date $t-1$ and $g_{Y_{n,t}}$ is the growth rate of the sectoral real output for n at date t. This definition of the real GDP growth corresponds to the growth rate of the chain-linked Laspeyres quantity index.[13] So, in effect, we mimic the definition of the real GDP growth rate used by statistical agencies of Europe and Japan. [14] In order to match the model analysis with the results of the empirical analysis in Section 2.2, we define the growth rate of ALP as follows:

$$g_{ALP_t} = g_{V_t^*} - g_{L_t}.$$

Equilibrium

The competitive equilibrium is defined as usual. Given the prices, the household maximizes their utility, all firms solve their maximization problem, and all the markets clear for all dates. We derive the equations which characterize the equilibrium.

The household's optimality conditions are

$$\frac{C_{t+1}}{C_t} = \beta\left(\frac{r_{i,t+1} + (1-\delta)\,p_{i,t+1}}{p_{i,t}}\right), \tag{5}$$

[13] Strictly speaking, the growth rate of the chain-linked Laspeyres index is based on actual growth rates, not log-difference of them. The numerical difference between the two methods of computing the real GDP growth rate is virtually zero.

[14] The US uses the Fisher quantity index for the real GDP. When we analyze the US, we define the real GDP as follows, $g_{V_t^*} \equiv \sum_{n\in\mathcal{N}} [s_{n,t} + s_{n,t-1}] \frac{1}{2} g_{Y_{n,t}}$. Along the BGP, this definition of the real GDP growth coincides with one by equation (4) since the nominal value added shares stay constant, $s_{n,t} = s_{n,t-1}$. (See Proposition 1.)

for all $i \in \mathcal{I}$. The transversality condition for the household's optimization is[15]

$$\lim_{t\to\infty} \beta^t \sum_{i\in\mathcal{I}} \frac{r_{i,t} + (1-\delta)\, p_{i,t}}{C_t} K_{i,t} = 0. \tag{6}$$

The firms' optimality conditions are

$$\alpha\theta_i p_{n,t} Y_{n,t} = r_{i,t} K_{i,n,t} \tag{7}$$

$$(1-\alpha)\, p_{n,t} Y_{n,t} = w_t L_{n,t}. \tag{8}$$

These two equations imply that in an equilibrium,

$$p_{n,t} = \frac{1}{A_{n,t}} \prod_{i\in\mathcal{I}} \left(\frac{r_{i,t}}{\theta_i \alpha}\right)^{\theta_i\alpha} \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} \tag{9}$$

$$\frac{K_{i,n,t}}{L_{n,t}} = \frac{\alpha}{1-\alpha}\theta_i \frac{w_t}{r_{i,t}} \tag{10}$$

Equation (9) comes from the fact that in a competitive good and factor markets, the marginal cost is equalized to the marginal revenue. Equation (10) comes from the fact that the firms have the identical CRS production functions up to their productivity.

Equation (9) immediately implies that the inverse of the relative price of good i reflects the TFP of sector i relative to one of the consumption good sector.

$$p_{i,t}^{-1} = A_{i,t}/A_{C,t}. \tag{11}$$

According to this model, the increase of the growth rate of $p_{i,t}^{-1}$ implies technological progress of sector a relative to the consumption good sector. So this model gives a foundation why the inverse of the relative price of asset class a can be a proxy for the technology level in Section 2.

There are final goods, labor, and capital markets in this economy. In an equilibrium, the following market clearing conditions are satisfied for all $t \geq 0, i \in \mathcal{I}$, and $n \in \mathcal{N}$:

$$Y_{C,t} = C_t \tag{12}$$

$$Y_{i,t} = K_{i,t+1} - (1-\delta_i)\, K_{i,t} \tag{13}$$

$$L_t = \sum_{n\in\mathcal{N}} L_{n,t} \tag{14}$$

$$K_{i,t} = \sum_{n\in\mathcal{N}} K_{i,n,t}. \tag{15}$$

[15] See Kamihigashi (2002) for a proof of necessity of equation (6) for the household's optimization problem.

A couple of the prices and allocations $\left\{(r_{i,t}, p_{i,t}, w_t)_{t\geq 0, i\in\mathcal{I}}, (K_{i,n,t}, L_{n,t}, K_{i,t}, C_t, Y_{n,t})_{i\in\mathcal{I}, n\in\mathcal{N}, t\geq 0}\right\}$ is an equilibrium if and only if the couple satisfies equation (3), (5), (6), (7), (8), (12), (13), (14), and (15).

Some of the analysis below study a balanced growth path (BGP) of the economy, which is a particular type of competitive equilibria. In order to define BGP, we start by assuming that the sectoral TFP grow at constant rates: for all $n \in \mathcal{N}$,

$$g_{A_{n,t}} = g_{A_n}.$$

Under the assumption that the sectoral TFP grow at constant rates, a balanced growth path (BGP) is a competitive competitive equilibrium where all the variables grow, but not necessarily equal, rates. To economize notation let g_X denote the growth rate of the variable X along the BGP.

3.2 Generalized Growth Accounting Equations

Now we are ready to estimate the sectoral TFP by using the equilibrium conditions we identified. Unlike the canonical growth model, it does not suffice to compute the (aggregate) Solow residual for this economy since we have more than one sector. So, we use the relative investment prices (11).

We begin by inspecting what the aggregated Solow residual corresponds to in our economy. Let g_{A_t} denote the Solow residual aggregated by using the Domer weights, $s_{n,t-1}$.

$$g_{A_t} = \sum_{n\in\mathcal{N}} s_{n,t-1}\left(g_{Y_{n,t}} - \alpha \sum_{i\in\mathcal{I}} \theta_i g_{K_{i,n,t}} - (1-\alpha)\, g_{L_{n,t}}\right).$$

By using equation (8) and (10), the aggregated Solow residual, g_{A_t}, can be written as follows:

$$g_{A_t} = g_{V_t^*} - \alpha \sum_{i\in\mathcal{I}} \theta_i g_{K_{i,t}} - (1-\alpha)\, g_{L_t}. \tag{16}$$

Notice that g_{A_t} is now expressed in terms of the observables and the production parameters, $\left(\alpha, (\theta_i)_{i\in\mathcal{I}}\right)$. which can be observable Because of the functional form assumption of the production technology (3), the Solow residual corresponds to the aggregated sectoral TFP by using the shares in value added as its weight.

$$g_{A_t} = \sum_{n\in\mathcal{N}} s_{n,t-1} g_{A_{n,t}}. \tag{17}$$

For the second equality, we use equation (3). Recall that the relative investment prices (11) reflect the sectoral TFPs. Taking log-difference of equation (11), the growth rate of the relative investment price of asset class $i \in \mathcal{I}$ is given by

$$-\ln g_{p_{i,t}} = g_{A_{i,t}} - g_{A_{C,t}}. \tag{18}$$

If we can compute g_{A_t}, then we have $A+1$ equations ((17) and (18)) and $A+1$ sectoral TFP growth rates, $\{g_{A_{n,t}}\}_{n\in\mathcal{N}}$. So, we can solve these linear equations and obtain our estimate of $\{g_{A_{n,t}}\}_{n\in\mathcal{N}}$ for each date.

3.3 Properties of Real Macroeconomic Variables Along BGP Characterization

In the empirical analysis below, we mainly use the properties of the BGP of this model. In this section, we derive the properties along the BGP. The following proposition summarizes the properties of the economy along the BGP.

Proposition 1. *Suppose that: (i) the sectoral TFP and labor grow at constant rates, $g_{A_{n,t}} = g_{A_n}$ and $g_{L_t} = g_L$; and (ii) for all $i \in \mathcal{I}$, the following inequality holds:*

$$g_{A_i} + \frac{\alpha}{1-\alpha}\sum_{i\in\mathcal{I}} \theta_i g_{A_i} + g_L > \ln(1-\delta_i). \tag{19}$$

Then along the BGP, we have:

1. *the shares in value added, $(s_n)_{n\in\mathcal{N}}$, coincide with the shares in the employment, $(L_{n,t}/L_t)_{n\in\mathcal{N}}$, and given by*

$$s_i = \alpha \frac{\exp(g_{A_i} + \alpha g_k + g_L) - (1-\delta_i)}{\beta^{-1}\exp(g_{A_i} + \alpha g_k + g_L) - (1-\delta_i)}\theta_i \quad i \in \mathcal{I}, \quad s_C = 1 - \sum_{i\in\mathcal{I}} s_i; \tag{20}$$

2. *the growth rate of ALP and the real wage are*[16]

$$g_{ALP} = \sum_{n\in\mathcal{N}} s_n g_{A_n} + \alpha g_k, \tag{21}$$

$$g_w = g_{A_C} + \alpha g_k, \tag{22}$$

where g_k is given by

$$g_k = \frac{1}{1-\alpha}\sum_{i\in\mathcal{I}} \theta_i g_{A_i}. \tag{23}$$

[16] The natural rate of interest, r^*, gets ample attention from the literature. In this model, the natural rate of interest, r^*, is determined only by the supply side of the economy and equal to the growth rate of the consumption, g_C. It turns out that the real wage growth along the BGP satisfies $g_w = \lambda + r^*$ for some $\lambda \in \mathbb{R}$. So any implication for the real wage holds automatically for the natural rate r^*. See Gourio and Rognlie (2020) which analyze the implication for the natural rate r^* when the demand side plays a role.

Proof. See Appendix B. □

The LHS of inequality (19) corresponds to the growth rate of the real capital stock of asset class a. So, inequality (19) requires that all the real stocks of capital grow at least rate $\ln(1-\delta_a)$,which is negative. This assumption is practically vacuous since it is often the case that the sectoral TFP and the labor input grow, not decline. Inequality (19) is used for showing that the nominal value-added shares are well-defined; namely s_n is strictly positive for all $n \in \mathcal{N}$ and adds up to one.

Our model is a multi-sector extension of the canonical growth model. So, to understand this proposition intuitively, first consider a version of the economy in which there exists only one investment good. Let g_{A_I} denote the growth rate of TFP for the investment good sector.[17] In such an economy, the growth rate of ALP is

$$g_{ALP} = \underbrace{s_C g_{A_C} + (1-s_C)\, g_{A_I}}_{\text{Direct Effect}} + \underbrace{\alpha \frac{1}{1-\alpha} g_{A_I}}_{\text{Capital Deepening Effect}} . \tag{24}$$

In this simpler economy, ALP grows because of the two effects: the direct effect from technology improvement and the indirect effect through capital deepening. The direct effects from the technology improvements, (g_{A_C}, g_{A_I}), are captured by the first two terms in equation (24). As the share in value added of sector n becomes larger, the (direct) impact of technological improvement in that sector sector n gets larger. The shares in value added appear in the first term in equation (24) because of our choice of the definition of the real GDP growth rate. (see equation (4)).[18] Since the consumption share in our sample of a typical country is around 75%, the direct effect is mostly governed by TFP of the consumption good sector, g_{A_C}. The last term in equation (21) represents the multiplier effect of capital accumulation on ALP growth. An increase in productivity, g_I, raises capital, which leads to increase GDP by αg_I. Consequently, the increase of GDP leads to more capital, which leads to more GDP, $\alpha^2 g_I$, and so on. The cumulation of this feedback loop is $\alpha/(1-\alpha)\, g_I$. This network-style feedback effect has a sizable role in our quantitative analysis.

When there are many capital goods in the economy, the two effects above are generalized as follows. The direct effect is trivially extended to $\sum_{n\in\mathcal{N}} s_n g_{A_n}$. As in the simpler version of the economy, the direct effect is mostly determined by the TFP growth rate of the consumption sector, C. To study the capital deepening effect in an economy with many capital goods, it is convenient to define a capital service index, which is a generalization of the real capital stock in the single investment good economy. Define the aggregate capital service index K_t as the geometric average of

[17] By using our notation, $\mathcal{I}$ becomes a singleton, $\mathcal{I} = \{I\}$.

[18] If we define the real GDP growth differently, the direct effect does not take the same form. For example, Greenwood et al. (1997) define the real GDP as the total nominal value added divided by the consumption price $p_{C,t}$, then the ALP growth rate along the BGP is now $g_{A_C} + \alpha g_k$. So the direct effect is g_{A_C}, not the weighted average of the sectoral TFP growth rates.

the real capital stocks: $K_t = \prod_{i\in\mathcal{I}} K_{i,t}^{\theta_i}$. It turns out that the growth rate of the capital service index to labor input corresponds to g_k, (23) along the BGP. Like the single investment good economy, an increase in productivity, g_{A_a}, raises the capital service, which leas to increase GDP by $\alpha\theta_i$, not α. The parameter θ_i governs how important asset class a for production. The rest is the same as in the single investment good economy. The increase of GDP leads to more capital, which leas to more GDP, $\alpha^2\theta_i$. The cumulative effect from g_{A_i} on GDP is $\theta_i\alpha/(1-\alpha)\, g_{A_i}$. So, the total cumulative effect from $\{g_{A_i}\}_{i\in\mathcal{I}}$ is $\sum_{i\in\mathcal{I}} \theta_i\alpha/(1-\alpha)\, g_{A_i}$, which is αg_k.

In the empirical application below, we are interested how the changes of the sectoral TFP affect the aggregate ALP growth rate. For this purpose, it is useful to take the total derivative of g_{ALP}.

$$\partial g_{ALP} = \sum_{n\in\mathcal{N}} s_n \partial g_{A_n} + \frac{\alpha}{1-\alpha}\sum_{i\in\mathcal{I}} \theta_i \partial g_{A_i} + \sum_{m,n\in\mathcal{N}} \left[\frac{\partial s_n}{\partial g_{A_m}} g_{A_n}\right] \partial g_{A_m}. \tag{25}$$

The first and second term in equation (25) correspond to the direct effect and the capital deepening effect. As mentioned above, the growth rate of ALP increases due to these effects. The last term in equation (25) is a composition effect. When a sectoral TFP growth rate changes, then equation (20) implies that the value added shares also change accordingly. The composition effect in equation (25) is second-order since it involves the term of multiplication of $\partial s_n/\partial g_{A_m}$ and g_{A_n}. So, when we conduct the empirical exercises in Section 5, we use equation (25) ignoring the last composition effect.[19]

3.4 Remarks on Modeling Assumptions

We introduced a model with a number of assumptions on the functional form. In this section, we discuss three noteworthy aspects of our modeling assumptions.

First, the growth accounting can be done under the general CRS function. In fact, the KLEMS dataset has the Solow residuals obtained under the general CRS sectoral production functions. Practically speaking, however, there is almost no difference between the Solow residuals obtained by using the general CRS production functions and ones obtained under the Cobb-Douglass functional form assumption. In this sense, there is little practical advantage in considering the general CRS function for growth accounting.

Second, we assume that the economy is characterized by a BGP, which requires that the production functions take the form of CRS Cobb-Douglass. This is the classic result obtained by Uzawa (1961).[20] It is possible to analyze the dynamics of the economy itself without assuming that the

[19] In Appendix **??**, we demonstrate that the partial derivatives, $\partial s_n/\partial g_{A_m}$, are sufficiently close to zero under reasonable parameters. Moreover, we confirm that the results obtained in Section 4.3 and 5 for the US are virtually identical to ones obtained by explicitly taking into account the composition effect.

[20] Grossman et al. (2017) endogenize human capital investment and show that, under certain substitutability be-

economy is not on the BGP. However, such an analysis requires us to estimate more parameters (e.g. various effective tax rates), which complicates our analysis. So, in this paper, we make the simplifying assumption on the production functions, and focus our analysis on a BGP.

Finally, we suppose that all production functions have the same factor shares, which is not necessary even for an equilibrium to have a BGP.[21] In reality, different sectors have different capital shares. While it is easy to build a model that allows for different capital shares, there is a practical difficulty. It is not obvious how the capital shares of different industries should be estimated. The industries we consider produce only consumption goods or investment goods, while actual industries produce both goods. Therefore, we are not able to link the industry in our model to the industry in the actual data. Our strong assumption assumes away this practical difficulty.

One way to proceed is to build a model that incorporates more general industry linkages which is compatible with the KLEMS sector classification. In order to do such an analysis, we need to have additional data such as the input-output tables compatible with the KLEMS dataset. Unfortunately, such a dataset is only available for few countries (e.g. Japan). Since not all the countries report these information, we can only analyze the economy with such an augmented model for the subset of the countries. This exercise is currently left for future research.

4 Estimation

In this section, we estimate sectoral productivities by using some of the equilibrium conditions obtained in Section 3. This section begins by calibrating the deep parameters which are needed in order to estimate the sectoral productivity sequences. Then by using the estimated productivity sequences, we check whether our model can account the ALP and capital dynamics in the data. On the top of these internal validation exercises, we provide an external validation by using wage data.

4.1 Prerequisite

In order to estimate the sectoral TFP growth rates, we need to specify: (1) the value-added shares $\{s_{n,t}\}_{n\in\mathcal{N},t}$; (2) the capital share α; (3) the shares in rental cost, $\{\theta_i\}_{i\in\mathcal{I}}$; and (4) the aggregated Solow residual g_{A_t} (16). The shares in value added, $\{s_{n,t}\}_{n\in\mathcal{N},t}$, are obtained by the KLEMS datasets and/or national accounts. The KLEMS dataset provides its time-series estimate of the

tween raw labor and human capital, BGP can exist without a Cobb-Douglass production function. Growth accounting using the model would require a variety of data on human capital in addition to the usual data and substitution parameters between production inputs. Such an extension would be interesting, but it is beyond the scope of the paper.

[21]This property only holds when the representative household's date-t payoff function takes the form of Cobb-Douglass. For a general analysis under CES, see Acemoglu and Guerrieri (2008).

capital share for each country, and we set the capital share α to the average value of it.

Unfortunately, the rental rate shares, $\{\theta_i\}_{i\in\mathcal{I}}$ is often unavailable from the KLEMS and national accounts, and very limited countries (e.g. Japan and the US) report their estimate of the shares in rental cost. Because of this unavailability, we employ the indirect method to estimate the rental rates following Gourio and Rognlie (2020).[22] Along the BGP, the rental rate shares are expressed in terms of the capital and investment shares:

$$\theta_i = \left(1 - \frac{s^I}{\alpha}\right) s_i^K + \frac{s_I}{\alpha} s_i^I, \tag{26}$$

where s^I is the share of nominal investment in GDP, s_a^I is the share of nominal investment of asset class a in the total nominal investment, and s_i^K is the share of nominal capital stock of asset class a in the total nominal capital stock. See Appendix F.3 for the derivation of equation (26). Since the nominal shares $\left\{s_i^I, s_i^K\right\}_{i\in\mathcal{I}}$ are obtained from national accounts, then we can use equation (26) to infer the rental rate shares $\{\theta_i\}_{i\in\mathcal{I}}$. [23] Finally since the KLEMS dataset have the growth rate of the real capital stock for each asset class, $g_{K_{i,t}}$, and labor input, g_{L_t}, the aggregated Solow residual g_{A_t} is computed by (16) given $\left(\alpha, (\theta_i)_{i\in\mathcal{I}}\right)$.

Note that the countries can have different shares in the value-added, the rental rate shares and so on. In order to avoid notational clutter, we drop an index representing a country c unless confusing.

Figure 6 shows the estimated $\left(\alpha, (\theta_i)_{i\in\mathcal{I}}\right)$. It is clear from Figure 6 that the parameters vary greatly across the countries. For example, the US capital share is 43% in the KLEMS dataset, but the capital share of Japan is 27%, the smallest among our sample countries. The US capital share α is higher than 1/3 which is the conventional value used in the literature, but not too high compared to 39%, which is the average US capital share after 1995 reported by PWT.[24] The share in rental cost for computing equipment (IT) is relatively small (3%), and the share for structure is the biggest for all the countries.

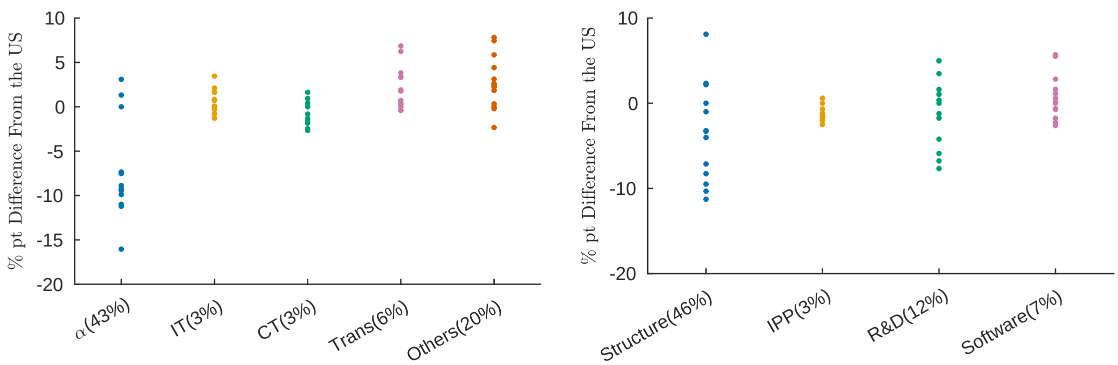
[22] Whelan (2003) uses another steady state moments in order to determine $\{\theta_a\}_{a\in A}$. In particular, Whelan (2003) matches the average nominal GDP shares and the real interest rate estimated by King and Rebelo (1999). A similar approach is taken by Greenwood et al. (1997).

[23] While this approach to measuring the rental rate shares is easy to implement, it requires an assumption that the economy is approximately on the balanced growth path. In Section 6.4, we compare our model-based rental rate shares with the estimates of capital shares for Japan (provided by JIP) and US (provided by BLS-BEA). They are quantitatively similar, and we redo our analysis based on the direct estimates of the rental rates for these two countries. See Section 6.4.

[24] We compute our benchmark capital share α for the US by using the capital share for the total economy reported in the BEA-BLS integrated industry-level Production Account. If we consider the market economy, not the total economy, the average capital share is reduced to 39%.

Figure 6: Estimated Parameters

(a) Capital Share and Equipment Shares (b) Non-Equipment Shares



Notes: Each dot represents an estimate of a parameter for a country. All values are expressed as the percentage point difference from the US, and the US estimate is shown in parentheses in the x-axis label. IT represents computing equipment, and CT is an abbreviation of the communication technology equipment.

4.2 Estimation of Sectoral TFPs Using Relative Prices

Having specified all the parameters, we can now estimate the sectoral TFP by solving equation (17) and (18). For all $n \in \mathcal{N}$,

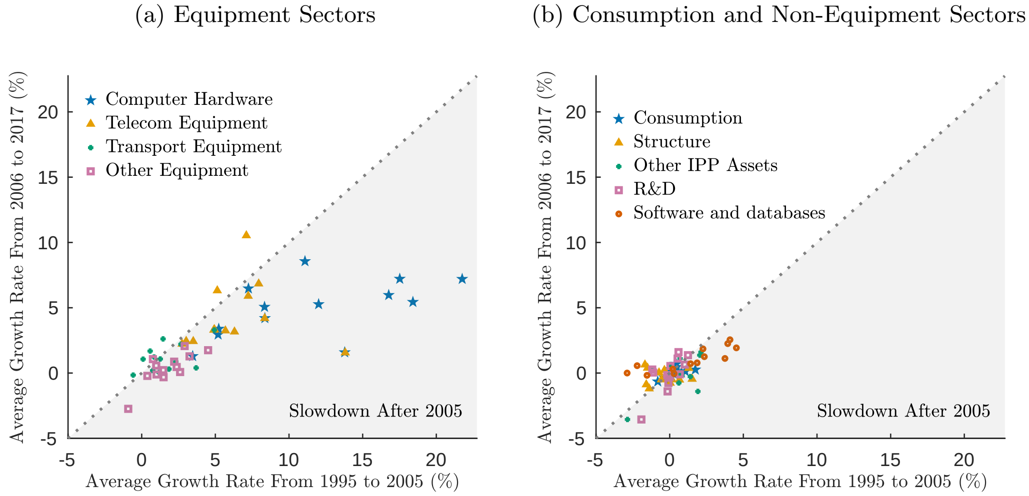
$$g_{A_{n,t}} = g_{A_t} + \sum_{m \in \mathcal{N}} s_{m,t-1} g_{p_{m,t}} - g_{p_{n,t}}.$$

In Figure 7, we display the estimated sectoral TFPs. As indicated by Figure 5c and 5d, Figure 7a and 7b imply that: the sectoral TFP growth rates of the equipment goods are much faster than those of non-equipment goods; the sectoral TFP growth rates of the non-equipment goods do not exhibit slowdown; and the TFP growth rate of the computing equipment experienced the most severe slowdown. For the US, the growth rate of the computing equipment declined by around 15%. These results basically reflect the movements of the relative investment prices.

4.3 Validation

Before turning to quantification of technological stagnation, we aim to give credibility to the basic model predictions. First, as an internal validation exercise, we confirm that our model can successfully reproduce the decline in the average productivity growth rate of each country after 2005. Second, as an external validation exercise we verify that the model-implied wage growth rate change predicts the associated actual real wage growth rate change across the countries.

Figure 7: Estimated Sectoral TFP



Notes: The vertical axis represents the average productivity growth rate from 1995 to 2005, and the horizontal axis represents it after 2005. Each dot represents a country.

4.3.1 About Dynamics

In order to conduce these validation exercises, we need to specify how the dynamics of the economy works. We assume that the economy is characterized by the BGP. Until 2005, the growth rate of the sectoral TFP is assumed to be constant, and the economy is on the BGP. After 2005, the growth rate of the sectoral productivity shifts unexpectedly, and the economy is assumed to jump to the new BGP instantaneously. As stated above, we choose year 2005 following Byrne et al. (2016) and Syverson (2017). We abstract from any dynamic adjustment of the economy. The bold assumption that the economy is always on the BGP allows us to skip to estimate many parameters, such as various effective tax rates, and makes a cross-country analysis clearer and easier.

The change in the TFP growth of good n after 2005 is specified as follows:

$$\mathbf{d}g_{A_n} = \frac{1}{11}\sum_{t=1995}^{2005} g_{A_{n,t}} - \frac{1}{12}\sum_{t=2006}^{2017} g_{A_{n,t}}.$$

In the following validation exercises, we use these shifts of the sectoral TFP growth rates.

We also need to specify the nominal GDP shares, s_n. The nominal GDP share is assumed to be averaged over the entire sample period. The GDP shares are stable over time, and do not affect our validation exercises and the main results by the reason explained after equation (25).[25]

[25]This procedure implicitly assumes that the depreciations are chosen so that the shares in value added match with those of data.

4.3.2 Internal and External Validation Exercises

We examine the estimated shifts of the sectoral TFP can explain the observed decline of the ALP growth rate across the countries. For this exercise, we use equation (25) and obtain the model-implied change of the ALP growth rate as follows:

$$\mathbf{d}g_{ALP} = \sum_{n\in\mathcal{N}} s_n \mathbf{d}g_{A_n} + \frac{\alpha}{1-\alpha}\sum_{i\in\mathcal{I}} \theta_i \mathbf{d}g_{A_i}.$$

We compare these model-implied change of the ALP growth rates with the empirical counterparts:

$$\mathbf{d}g_{ALP}^{\text{Data}} = \frac{1}{11}\sum_{t=1995}^{2005} g_{A_{ALP,t}} - \frac{1}{12}\sum_{t=2006}^{2017} g_{A_{ALP,t}}. \tag{27}$$

Note that the internal validation exercise is not trivial. This is because we assume that: (1) the economy is characterized by the BGP and; (2) and all the growth rates of the endogenous variables are expressed in terms of the estimated sectoral TFP growth rates. If other shocks affect the economy, the internal validation can fail. For example, if the Japanese financial crisis triggered a long-lasting decline of the Japanese ALP without affecting the Solow residuals, then the change of the ALP growth rate of Japan cannot be reproduced. That is, the internal validation fails. Also if the economy is not well-approximated well by the BGP, then the internal validation can fail too.

On the top of the internal validation, we also provide two external validation exercises. That is, we examine whether the model explains the moment which are not used in our TFP estimation procedure. In the first exercise, we examine whether the model can explain the change of the real wage growth rate in our data before and after 2005.[26] Taking the total derivative of equation (22) and using the estimated changes of the sectoral TFP, the model-implied change of the real wage growth rate is

$$\mathbf{d}g_w = \mathbf{d}g_{A_C} + \frac{\alpha}{1-\alpha}\sum_{a\in\mathcal{I}} \theta_a \mathbf{d}g_{A_a}. \tag{28}$$

We compare the mode-implied real wage growth with the empirical counterpart, which is given by

$$\mathbf{d}g_w^{\text{Data}} = \frac{1}{11}\sum_{t=1995}^{2005} g_{w_t} - \frac{1}{12}\sum_{t=2006}^{2017} g_{g_{w_t}},$$

where g_{w_t} is the growth rate of the real wage at date t.[27] In the second exercise, we check whether

[26] Hayashi and Prescott (2002) conduct a similar external validation exercise. They examine whether the model can explain the return on capital, r_t.

[27] The real wage growth rate is computed by subtracting the growth rate of the nominal wage from the growth rate of the consumption deflator. The nominal wage growth is computed by subtracting the growth rate of the labor compensation from the growth rate of labor input.

the model can explain the change of the capital service to labor growth rate. (Recall equation 23.)

$$\mathbf{d}g_k = \frac{1}{1-\alpha}\sum_{i\in\mathcal{I}}\theta_i \mathbf{d}g_{A_i}.$$

The empirical counterpart of $\mathbf{d}g_k$ is computed as follows:

$$\mathbf{d}g_k^{\text{Data}} = \frac{1}{1-\alpha}\sum_{i\in\mathcal{I}}\theta_i \left(\frac{1}{11}\sum_{t=1995}^{2005}\left(g_{K_{i,t}} - g_{L_t}\right) - \frac{1}{12}\sum_{t=2006}^{2017}\left(g_{K_{i,t}} - g_{L_t}\right)\right).$$

Recall variable, $g_{K_{n,t}}$, is the growth rate of capital stock of $i \in \mathcal{I}$.

In Figure 8a, we plot these model-implied growth rate changes, $\mathbf{d}g_{ALP}$, against the associated actual average ALP growth rate changes. In Figure 9b, we plot the model-implied wage growth rate changes, $\mathbf{d}g_w$, against the corresponding actual real wage changes. In Figure 9c, we plot the model-implied capital service to labor ratio agains the associated the empirical counterpart. Note that almost all the points in Figure 8a, 9b, and 9c are near the 45-degree line. So, these figures imply that the model can reproduce, on average, the changes of the ALP growth rates as well as some other growth rate changes across the countries.

5 Quantitative Analysis

This section begins with a quantitative analysis of the extent to which technological stagnation has pushed down the ALP growth rate. We show that the technological stagnation has a sizable and heterogeneous impact on the ALP growth rate across countries. Then, we proceed by examining this heterogeneity by conducting two additional simulations. Finally, we analyze the impact of this technological stagnation on other macroeconomic variables besides the ALP growth rate.

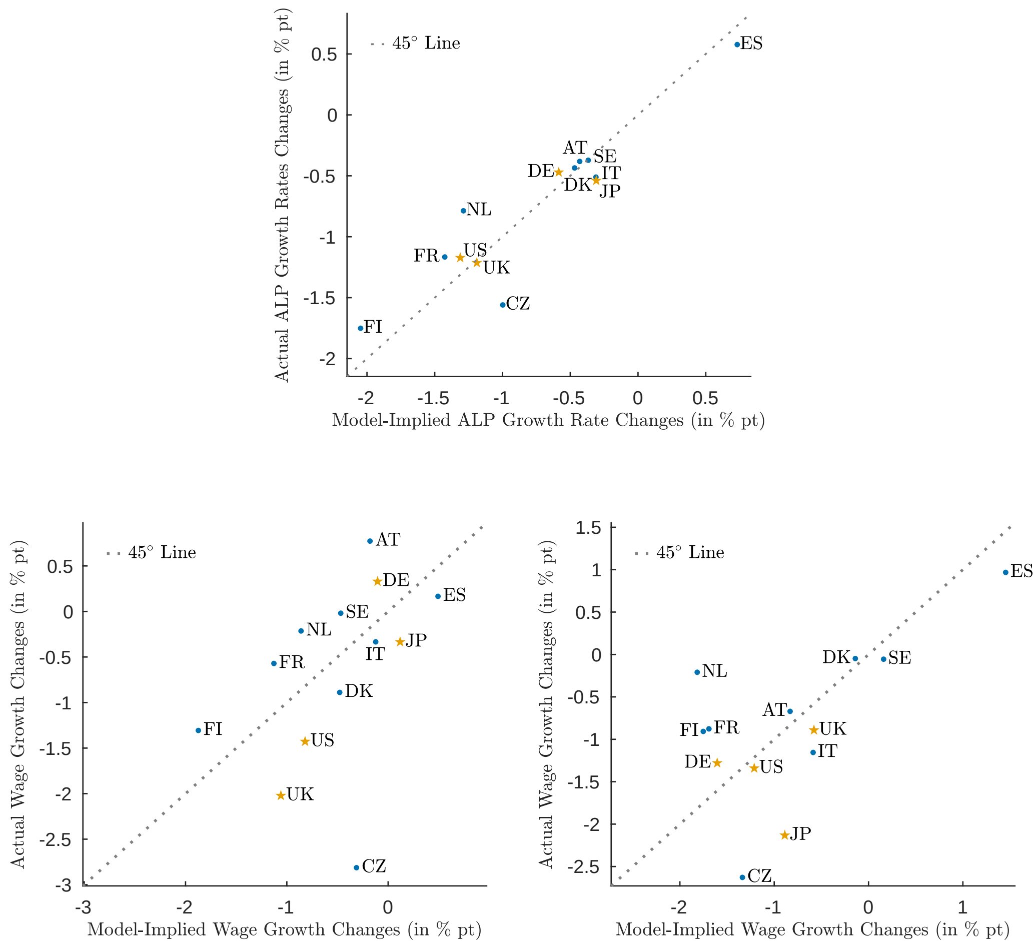
5.1 Tech-Induced Stagnation

We will examine the extent to which the decline in average labor productivity in the last decade can be explained by the technological stagnation. We interpret the declines of the TFP growth rates of the equipment sectors $\mathcal{E}$ as reflecting technological stagnation following Greenwood et al. (1997). We explore the implications of the technological stagnation on the ALP growth rate. To do so, we calculate model-implied declines of the ALP growth rates coming from technological stagnation in the equipment sectors as follows.

$$\mathbf{d}g_{ALP}^{\text{Tech}} = \sum_{i\in\mathcal{E}} s_i \mathbf{d}g_{A_i} + \frac{\alpha}{1-\alpha}\sum_{i\in\mathcal{E}}\theta_i \mathbf{d}g_{A_i}.$$

Figure 8: Validation Exercises

(a) Internal Validation (ALP)



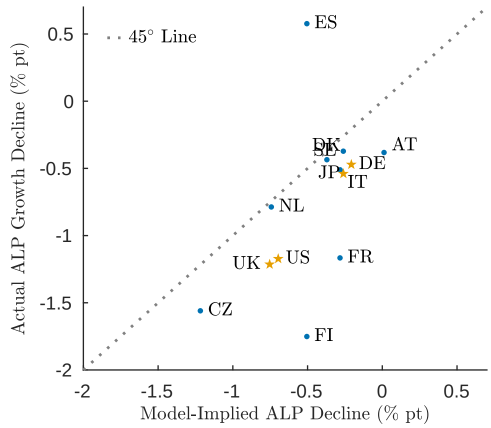
(b) External Validation (Real Wage) (c) External Validation (Capital Labor Ratio)

We then compare this model-implied ALP stagnation with the empirical counterpart, $\mathbf{d}g_{ALP}^{\text{Data}}$ (See equation (27)).

We report our results in Figure 9 and Table 1. Each point in Figure 9 represents a country. The vertical axises of Figure 9 plots the difference in the average growth rate of productivity before and after 2005, and the horizontal axises the difference in the average growth rate before and after 2005 predicted by the model.

There are two points worth mentioning about this result. The technological stagnation in the equipment sectors alone can largely explain the global declines of the ALP growth rates of ALP. This result is especially true for countries for the UK and the US (see Table 1). For example, the growth rate of the US ALP has declined by 1.17% since 2005, and the model predicts that the growth rate declines by 0.83% due to the technological stagnation, which is more than 70% of the total decline. In the case of the UK, the growth rate of ALP has declined by 1.21%, and the model

Figure 9: Quantification of Technology Stagnation



NOTEs: The two letters near the dots indicate the country represented by the ISO code, and the dotted line represents the 45° line.

predicts a decline in growth of 0.93%. The technological stagnation also induce lower growth for Japan and Germany albeit on a slightly smaller scale. For Japan, the technological stagnation explains around 60% of the total ALP growth decline, and for Germany, 40%.

In Table 1, we decompose the effects from the technological stagnation into pieces. For Japan and the US, the stagnation of computer technology has been the most important driver of stagnation in economic growth. For the US, the magnitude is largest among the developed countries.

This result for the US might sound surprising since the share of computing equipment in value added and the share in rental costs are both relatively small. To be specific, these share of the US are 1% for the value added share and 2.5% for the rental cost share. However the technology stagnation in the computing equipment sector is huge: the TFP growth rate of this sector declined by 14.5%. So the effect from the computing equipment sector (IT) on the aggregate ALP is

$$\left(\underbrace{s_{\text{IT}}}_{=.01} + \underbrace{\frac{\alpha}{1-\alpha}}_{=.75} \times \underbrace{\theta_{\text{IT}}}_{.025}\right) \times \underbrace{\mathbf{d}g_{ALP,\text{IT}}}_{-14.5\%} = -.42\%.$$

This calculation makes it clear that the capital deepening effect plays a big role for inducing this big negative effect from the computing equipment sector on the US ALP growth rate.

On the other hand, for Germany and the UK, the technological stagnation of other equipment sector has significantly lowered their ALP growth rate.

Table 1: Technological Stagnation and Its Decomposition

	$\mathbf{d}g_{ALP}^{\text{Data}}$	$\mathbf{d}g_{ALP}^{\text{Tech}}$	Decomposition of $\mathbf{d}g_{ALP}^{\text{Tech}}$			
			IT		Other Equipment	
			Weight on $\mathbf{d}g_a$	$\mathbf{d}g_a$	Weight on $\mathbf{d}g_a$	$\mathbf{d}g_a$
Austria	-0.38%	0.01%	0.02	-2.25%	0.18	0.3%
Czech Republic	-1.56%	-1.22%	0.06	-4.16%	0.35	-2.76%
Denmark	-0.44%	-0.37%	0.05	-3.28%	0.17	-1.24%
Finland	-1.75%	-0.5%	0.01	-10.31%	0.18	-1.97%
France	-1.17%	-0.28%	0.01	-0.78%	0.14	-1.91%
Germany	-0.47%	-0.21%	0.03	-1.85%	0.19	-0.85%
Italy	-0.51%	-0.28%	0.02	-2.17%	0.22	-1.11%
Japan	-0.54%	-0.26%	0.03	-6.74%	0.17	-0.42%
Netherlands	-0.79%	-0.74%	0.03	-12.97%	0.18	-1.81%
Spain	0.58%	-0.5%	0.02	-10.8%	0.14	-1.83%
Sweden	-0.37%	-0.26%	0.04	-2.53%	0.26	-0.6%
United Kingdom	-1.21%	-0.75%	0.03	-12.2%	0.16	-2.53%
United States	-1.17%	-0.7%	0.03	-14.56%	0.21	-1.33%

Notes: The first column shows the difference in ALP growth rate until 2005 and after 2005. The second column shows the effect of technological stagnation of the equipment sectors on the ALP growth rate after 2005.

5.2 Investigation of Heterogeneous Responses

We proceed by examining a cause of heterogeneous impacts of the technology stagnation. The effect of stagnation in the technological progress of equipment on ALP differs across countries because of the following differences: (1) the parameters of the production function $\left(\alpha, (\theta_i)_{i\in\mathcal{I}}\right)$ and the shares in value added $(s_n)_{n\in\mathcal{N}}$ and; (2) the magnitude of the shocks, $(\mathbf{d}g_{A_i})_{i\in\mathcal{E}}$. In order to isolate these effects on ALP and the magnitude of the shocks, we conduct the two experiments. In the first experiment, we calculate how much the ALP of country c has dropped relative to the US if country c experienced the technology stagnation in equipment of the same size as the US after 2005, $\left(\mathbf{d}g_{i,ALP}^{\mathbf{US}}\right)_{i\in\mathcal{E}}$. This relative decline of the ALP growth rate for country c is denoted by $\mathcal{D}^c$, which is given by

$$\mathcal{D}_1^c = \underbrace{\sum_{i\in\mathcal{E}}\left(s_i^c \mathbf{d}g_{i,ALP}^{\mathbf{US}} + \frac{\alpha^c}{1-\alpha^c}\theta_i^C \mathbf{d}g_{i,ALP}^{\text{US}}\right)}_{\text{Counterfactual ALP Decline of Country } c} - \sum_{i\in\mathcal{E}}\left(s_i^{\text{US}} \mathbf{d}g_{i,ALP}^{\mathbf{US}} + \frac{\alpha^{\text{US}}}{1-\alpha^{\text{US}}}\theta_i^{\text{US}} \mathbf{d}g_{i,ALP}^{\text{US}}\right),$$

where s_i^c is the nominal GDP share of investment good i, α^c is the capital share, and θ_i^c is the rental share of investment good i of country c. The first summation term is the counter-factual decline of the ALP growth rate of country c if the country experienced the same technology stagnation as the US. The second summation terms is the decline of the ALP growth rate of the US. The variable $\mathcal{D}^c$ summarizes the effects on the ALP growth rate coming from the parameter differences. To get more insights, we decompose $\mathcal{D}^c$ as follows:

$$\mathcal{D}_1^c = \underbrace{\sum_{i\in\mathcal{E}} \left(s_i^c - s_i^{\text{US}}\right) \mathbf{d}g_{i,ALP}^{\mathbf{US}}}_{=\mathcal{D}_s^c} + \underbrace{\left(\frac{\alpha^c}{1-\alpha^c} - \frac{\alpha^{\text{US}}}{1-\alpha^{\text{US}}}\right) \sum_{i\in\mathcal{E}} \theta_i^{\text{US}} \mathbf{d}g_{i,ALP}^{\text{US}}}_{=\mathcal{D}_\alpha^c} + \underbrace{\frac{\alpha^c}{1-\alpha^c} \sum_{i\in\mathcal{E}} \left(\theta_i^C - \theta_i^{\text{US}}\right) \mathbf{d}g_{i,ALP}^{US}}_{=\mathcal{D}_\theta^c}.$$

The first term $\mathcal{D}_s^c$ shows the impact of differences in the GDP shares $\{s_i\}_{i\in\mathcal{E}}$ on the growth rate of ALP. The second term $\mathcal{D}_\alpha^C$ shows the effect of differences in the capital share, α. The last term shows the effect of differences in the shares of rental costs.

In the second experiment, we calculate how much the US ALP growth rate would decline relatively if the US experienced a technological stagnation of the magnitude experienced by other countries.

$$\mathcal{D}_2^c = \sum_{i\in\mathcal{E}} \left(s_i^{\text{US}} \left(\mathbf{d}g_{i,ALP}^c - \mathbf{d}g_{i,ALP}^{\text{US}}\right) + \frac{\alpha^{\text{US}}}{1-\alpha^{\text{US}}} \theta_i^{\text{US}} \left(\mathbf{d}g_{i,ALP}^c - \mathbf{d}g_{i,ALP}^{\text{US}}\right) \right),$$

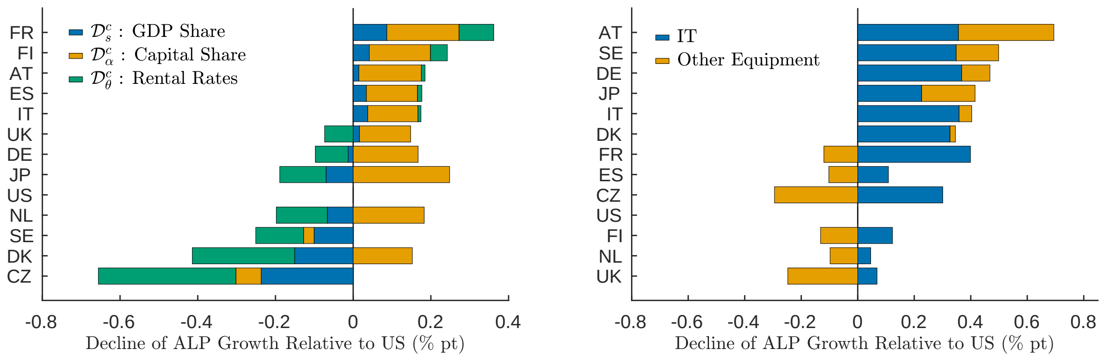
where $g_{i,ALP}^c$ is the change of the ALP growth rate of country c. The variable, $\mathcal{D}_2^c$, represents the effect of the magnitudes of the shocks on the ALP growth rate. If $\mathcal{D}_2^c$ is negative for many countries, then the US experiences greater technological stagnation than the other countries.

The results of the first experiment are displayed in Figure 10a and the second experiment in Figure 10b. There are three points to note about Figure 10a. The first is that the effect of different GDP shares on the ALP growth rate is not substantial, except for a few countries. This is due to the fact that consumption accounts for a large share of GDP. The second point is that $\mathcal{D}_\alpha^c$ is positive in many countries. This indicates that the US is more sensitive to technological stagnation because the US has a higher capital share (i.e. lower labor share) than other countries. Finally, the (overall) growth rate of ALP, $\mathcal{D}_1^c$, does not fall as much as in the US for many countries, even when they experience the same shocks as in the US This is because the negative effect caused by the difference in rental shares is offset by the positive effect caused by the difference in capital shares. So the overall effect on the ALP growth rate are positive except for Sweden, Denmark, and Czech Republic.

Figure 10b implies that the technological stagnation of computing equipment is the most severe in the US. This can be seen from the fact that the most blue bars are positive. The magnitude of the shocks of the UK is greater than one of the US, which is driven by the stagnation of other equipment and tele-communication equipment. In sum, Figure 10a and 10b imply that both channels play a

Figure 10: Two Experiments

(a) Role of Parameters (b) Role of The Magnitude of Shocks $(\mathbf{d}g_{A_n})_{n\in\mathcal{E}}$



NOTEs: Figure 10a shows how much the ALP growth rate would have declined compared to the US if each country had experienced the same technological stagnation as the US. Figure 10b shows how much the growth rate of ALP would have declined if the US had experienced the same technological stagnation as other countries. Figure 10b depicts the effect from capital good i, $s_i^{\text{US}}\left(\mathbf{d}g^c_{i,ALP} - \mathbf{d}g^{\text{US}}_{i,ALP}\right) + \frac{\alpha^{\text{US}}}{1-\alpha^{\text{US}}}\theta_i^{\text{US}}\left(\mathbf{d}g^c_{i,ALP} - \mathbf{d}g^{\text{US}}_{i,ALP}\right)$, for each asset class $i \in \mathcal{E}$. So, the total effect $\mathcal{D}^c_2$ is the sum of these four effects. In both figures, the value of the bar is normalized so that the ALP growth rate is zero if the US experiences its own technological stagnation, which is the benchmark. For example, the bottom bar in Figure 10b indicates if the US experienced the technological stagnation that the UK experienced, the ALP growth of the US would be 0.5 lower than the benchmark.

sizable role for the heterogeneous impacts of the technological stagnation.

5.3 Implications for Other Macro Variables

The technological stagnation affects the growth rate of other macroeconomic variables. Here we explore the implications for investment (and therefore capital) and the real wage.

Recently, Gutiérrez and Philippon (2017) point out that the US investment is weak relative to the level implied by Tobin's Q. One of the facts presented by Gutiérrez and Philippon (2017) is that the ratio of the net investment relative to net operating surplus, denoted by ι_t declined in the last decade for the US. We examine whether our model and the technological stagnation can speak to this fact quantitatively.

We start by defining the ratio ι_t as follows:[28]

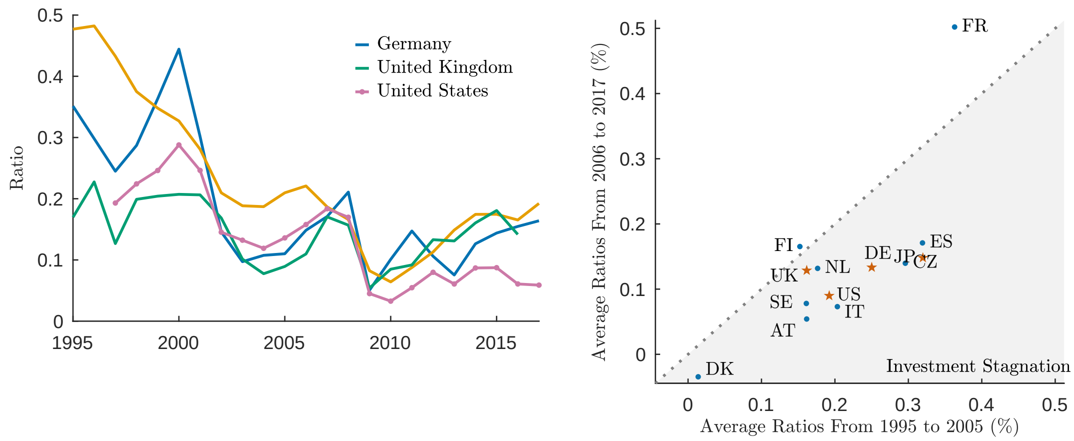
$$\iota_t = \frac{\sum_{i\in\mathcal{I}} p_{i,t} I_{i,t} - \sum_{i\in\mathcal{I}} \delta_i p_{i,t} K_{i,t}}{\sum_{n\in\mathcal{N}} p_{n,t} Y_{n,t} - \sum_{n\in\mathcal{N}} w_t L_{n,t} - \sum_{i\in\mathcal{I}} \delta_i p_{i,t} K_{i,t}}.$$

Figure 11a depicts how the ratios for Germany, Japan, the UK, and the US have evolved since 1995, and Figure 11b depicts the average ratios by 2005 for the horizontal axis and after 2005 for the vertical axis. As shown by Gutiérrez and Philippon (2017), the average ratio for the US declined

[28]The KLEMS dataset has an estimate of depreciation rate for each asset class $a \in \mathcal{I}$, and assumes that they are common across the countries. We use these depreciation rates for our construction of ι_t for each country.

Figure 11: Net Investment Relative to Net Operating Surplus

(a) Time-Series (b) Change



by 10% after 2005. Also, Figure 11b implies that all the countries except France and Finland experienced the declines of the ratios after 2005. The magnitude of the declines vary across the countries: the ratios of Germany and the UK fell as much experienced a similar magnitude of the decline as the

We move on to connects the ratio ι_t to the model primitives. We establish:

Proposition 2. *The ratio of the net investment relative to net operating surplus along the BGP is*

$$\iota = \sum_{i \in \mathcal{I}} \omega_i \times \left[\frac{\exp\left(g_{A_i} + \alpha g_k + g_L\right) - 1}{\beta^{-1} \exp\left(g_{A_i} + \alpha g_k + g_L\right) - 1} \right], \tag{29}$$

where ω_i is given by

$$\omega_i \propto \frac{\beta^{-1} \exp\left(g_{A_i} + \alpha g_k + g_L\right) - 1}{\beta^{-1} \exp\left(g_{A_i} + \alpha g_k + g_L\right) - (1 - \delta_i)} \theta_a, \quad \sum_{a \in \mathcal{I}} \omega_a = 1.$$

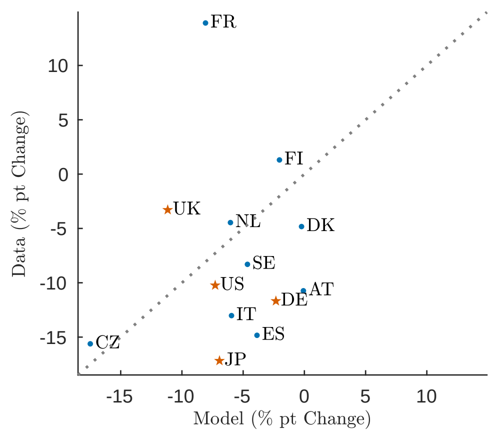
Proof. See Appendix 2. □

The weight ω_a in equation (29) represents the share of investment $a \in \mathcal{I}$ in the operational surplus along the BGP. The square bracket term in equation (29) represents the ratio of the net investment to the operation surplus associated with asset class $a \in \mathcal{I}$ along the BGP.[29] Given the equation (29), it is evident that conditional on the shares in the operational surplus, the ratio ι goes down when the sectoral TFP growth rates stagnate.

Using equation (29), we quantify the impact of the technological stagnation on the ratio of

[29] Mathematically, the ratio associated with asset class $a \in \mathcal{I}$ along the BGP is $\left(p_{a,t} I_{a,t} - \delta_a p_{a,t} K_{a,t}\right) / \left(r_{a,t} K_{a,t} - \delta_a p_{a,t} K_{a,t}\right)$.

Figure 12: Ratio of Net Investment to Operational Surplus



NOTEs: The horizontal axis represents the decline in the ratio implied by the model, and the vertical axis represents the decline of the average ratio ι before 2005

the net investment to the operational surplus. Note that we know all the parameters and the sectoral growth rate of TFP except for discount factor β. We choose discount factor β so that the average ratio of country i until 2005 matches exactly with the empirical counterpart of that country. Then with the calibrated discount factor, β, we compute the ratio when the equipment-specific technological stagnation happens while keeping the other growth rates fixed.

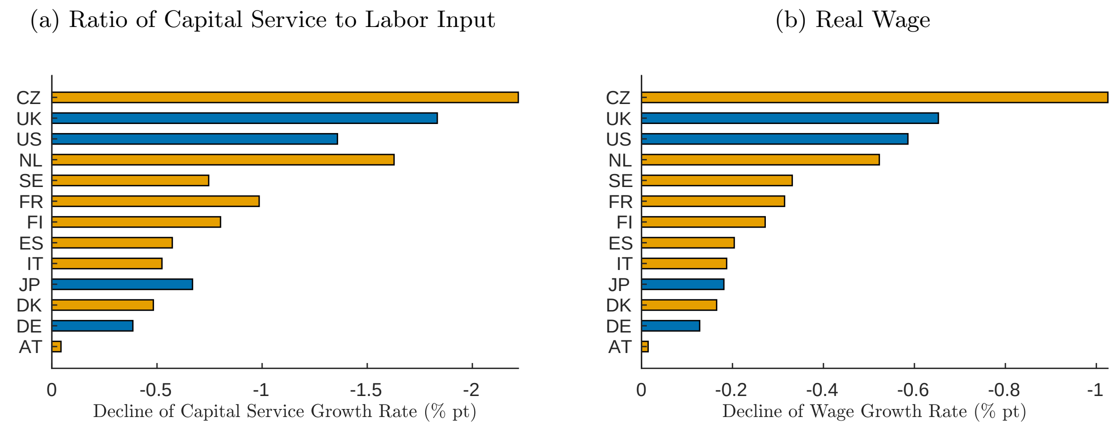
The results are summarized in Figure 12. The model implies substantial declines of the ratios across the countries. In particular, the equipment-specific technological stagnation weakens investment of the US. According to the model, the technological stagnation, $\{\mathbf{d}g_{A_i}\}_{i\in\mathcal{E}}$, induces 7.2% decline of the ratio ι. So, the model accounts for 70% of the decline of the ratio.

We move on to quantifying the effect of the technological stagnation on the growth rates of capital and real wage by using equation in Proposition 1. Figure 13a and 13b show the extent to which the growth rate of capital service g_k and the real wage, g_w, have declined due to the equipment-specific technological stagnation, $\{\mathbf{d}g_{A_i}\}_{i\in\mathcal{E}}$. The decline of g_k is observed across the countries, and this stagnation of capital induces the decline of the real wage across the countries. Note that this stagnation in the growth rate of real wages can have a large cumulative effect. For example, from 2005 to 2017, the technological stagnation lower the wage of Japan from 1000k yen to 976k if the economy has zero inflation on average. So during this period, the stagnation had driven down the wage by 2.4%.

6 Robustness

In this section, we provide additional evidence on our technological stagnation hypothesis. In particular, we address the problem of mismeasurement, and provide a different TFP estimation

Figure 13: Implications for Capital and Real Wage



NOTEs: Figure 13a shows the degree to which the growth rate of capital services has declined due to technological stagnation since 2005. Figure 13b shows the extent to which the growth rate of real wages has declined due to technological stagnation since 2005.

methods, which confirms that the developed countries' technology has stagnated. By using this different method, we can partially address whether our result is driven by the rise of China.

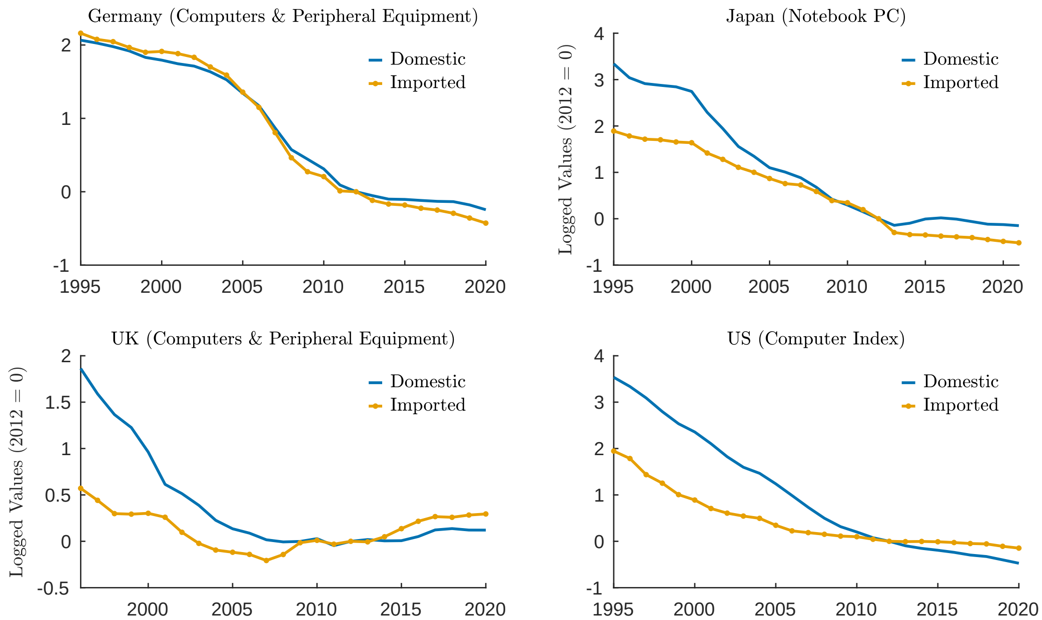
6.1 Measurement Problems

According to the model developed in Section 3, the relative investment price reflects the relative sectoral technology levels. So, the decline of the relative investment price implies that the investment-specific TFP grows faster than the TFP growth rate of the consumption good sector does. If the relative investment price stops falling, then the model implies the slowdown of the investment-related TFP growth rates.

However, there is another interpretation of the fact about the relative investment prices. The slowdown of the relative investment-good price decline may reflect mismeasurement by the statistical agencies. According to this view, true relative investment prices among the developed countries have kept declining all the time, but for some reasons, the amount of the mismeasurement have increased since around 2005. Due to mismeasurement, the relative equipment price indexes stopped falling in the last two decades. The mismeasurement arguments can be divided into two groups: composition and general mismeasurement problem. In this paper we provide evidence against the measurement arguments, and introduce several past papers which study the same issue.

The composition argument is based on a simple observation. Roughly speaking, the growth rate of an equipment price index is a weighted sum of the domestic ICT good price and the imported ICT good price, and the weight is the nominal investment share. For some reason, suppose that the imported computing equipment price index falls less than the domestic index does, but the share of the imported PCs increase over time. Then even if the price indexes fall at constant rates, the

Figure 14: PC Prices



NOTE: Because of the data availability, we do not have the prices of the same product. For example, we depict the price for notebook PC for Japan, but these prices are not available for the other countries. Thus, we pick a price of similar products or indexes. For Germany and the UK, we choose the price index for the computers and peripheral equipment. For the US, we choose the price index for computer.

growth rate of the aggregate computing equipment index can slow down due to the composition effect. This composition problem is first introduced by Byrne and Pinto (2015).

In order to see whether this composition effect stops the relative investment prices from falling, we study the behaviors of the domestic and imported computing equipment price or index for Japan, Germany, the UK and the US. Figure 14 depicts these domestic and imported prices.

Except for Germany, it is true that the imported computer price have fallen less than the domestic computer price. But the growth rates of the domestic and imported computer prices has slowed down substantially. So the composition plays a less role here. At least, for Germany, the composition plays no role since the domestic computer price coincides with the imported one.[30]

Byrne et al. (2016) also argues that the mismeasurement of ICT goods in the US is significant, but have happened before the slowdown of the US. Byrne et al. (2016) also examine the hypothesis that the slowdown reflects the growing importance of poorly measured industries with low productivity growth, such as health care and other services. Byrne et al. (2016) concludes that this

[30] When the statistical agencies compute their imported price sequences, they convert the foreign currencies into the domestic currency. So the exchange rate might derive the behavior. We provide evidence against this argument. Bank of Japan reports the price indexes based on the contract currency too. The imported computer price based on the contract currencies move exactly the same as the imported PC price based on yen. Therefore, at least for Japan, the exchange rate does not derive this result. See Figure 18 in Appendix G. Unfortunately, the other three countries do not report the price indexes based on the contract currency.

hypothesis cannot explain the decline of the ALP of the US too.

? also address the measurement hypothesis on the US slowdown. He discusses not only the problem of mismeasurement of investment goods prices, but also a wide range of mismeasurement problems related to output. **?** lays out various indirect evidence against the measurement hypothesis and concludes that "the measurement hypothesis faces a higher bar in the data, at least in terms of its ability to account for a substantial portion of the measured output lost to the productivity slowdown." Aghion et al. (2019) also analyze the problem of measurement, but from a different angle. Aghion et al. (2019) begin their analysis by noting that imputation procedure related with exiting products systematically causes measured real economic growth to be lower than the true economic growth. Aghion et al. (2019) propose a method correcting this bias, but could not explain the stagnation of the US growth rate after 2005. Given our evidence and the findings by the literature, we conclude that the decline of the relative investment price or slowdown of the economic growth do not come from problems of mismeasurement. [31]

6.2 Direct Estimate of Sectoral TFPs from KLEMS

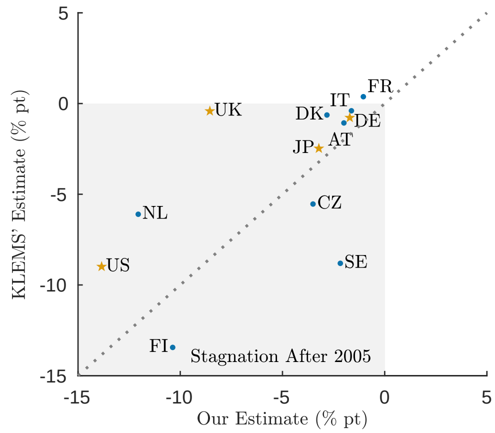
We used the relative prices of investment goods by asset class when estimating productivity. These relative prices of investment goods by asset class may be affected by imports and exports. Therefore, there is a concern that the changes in these relative prices do not reflect the changes in technology. On the other hand, the productivity estimated by KLEMS is less affected by imports and exports. This is because the KLEMS uses the growth accounting of the domestic production by industry. In this subsection, we compare the productivity of the KLEMS dataset with our productivity estimated by using the relative prices.[32]

Since this classification of industries in the productivity estimation in the KLEMS dataset does not perfectly match the classification of the asset classes. So we choose to compare the TFP growth rate of the computer sector (C26) with the growth rate of the computing hardware. In Figure 15, we graph our estimate of productivity declines of the computing equipment and the corresponding KLEMS's estimate. Figure 15 shows that both estimates show stagnation since almost all the dot lies inside of the shaded area. So, while the magnitude of the declines differ across the methods,

[31] As argued by Byrne et al. (2016), there is no doubt that the measurement issues have been hugely problematic for ICT goods. Moreover, measurement of macroeconomic variables are becoming increasingly more difficult due to the shift to services and cloud computing in the economy in general.

[32] You may be wondering why we do not use KLEMS data for our analysis in the first place. With KLEMS data alone, it is unknown whether the output of an industry is used as a consumption good or as an investment good. As is clear from Proposition 1, such a distinction is critically important when drawing the implications for growth. In order to make this distinction, we need an input-output (IO) table compatible with the KLEMS sector classification. Unfortunately not all countries report such an IO table as mentioned in Section 3.4. This is why we chose to use the investment prices in order to estimate the TFP growth rates. Such an IO tables are available for Japan, Germany, the UK, and the US. So, we are currently conducting a detailed analysis using KLEMS data and the compatible IO tables without using investment goods prices.

Figure 15: TFP Comparisons



Notes: The horizontal axis shows the extent to which the growth rate of our productivity for the computing equipment has decreased before and after 2005, $\mathbf{d}g_{A_{\text{Computer}}}$. The vertical axis shows how much the productivity growth rate of KLEMS data for sector C26 (Computer, electronic and optical products) has decreased before and after 2005.

we confirm that we observe the same technological stagnation in the KLMES dataset. The fact that the same stagnation is observed in the KLMES dataset implies that our result is not entirely driven by trade.

6.3 Rise of Market Power

Our empirical approach does not allow us to measure changes in demand or market power separately from technology. Therefore, there is a lingering doubt that the slowdown in the decline of relative prices of investment goods may be due to changes in market power or demand. To overcome this doubt, we attempt to estimate changes in technology and market power independently using the approach by Hall (2018) in this subsection. [33] Moreover, we supplement our discussion with De Loecker and Eeckhout (2021) measuring markup in various countries.

When the producers have a market power, the marginal cost is not equal to the price. So, equation (9) does not hold. Instead, equation (9) is generalized as follows:

$$p_{i,t} = \frac{\mu_{i,t}}{A_{i,t}} \prod_{i \in \mathcal{I}} \left(\frac{r_{i,t}}{\theta_i \alpha} \right)^{\theta_i \alpha} \left(\frac{w_t}{1-\alpha} \right)^{1-\alpha},$$

where $\mu_{i,t}$ is the markup charged by the representative firm in sector i at date t. The above equation implies that the relative price change is a function of relative TFP change and relative markup change:

$$\Delta \ln p_{i,t} - \Delta \ln p_{C,t} = \Delta \ln A_{C,t} - \Delta \ln A_{i,t} + \Delta \ln \mu_{i,t} - \Delta \ln \mu_{C,t}.$$

[33] Another approach is to measure the markup by the method used in De Loecker et al. (2020). A difficulty of this approach is that we do not have reliable capital inputs firm-level data. As discussed by Hall (2018), the KLEMS datasets have an advantage on this aspect.

Therefore, the increase of the markup charged by equipment sectors can slowdown the relative price decline.

Hall (2018) begins by relaxing the perfect competition assumption by allowing firms to have market power. The sectoral Solow residual, $s_{i,t}$, is generalized as follows:

$$\Delta \ln s_{i,t} = \mathcal{L}_{i,t} \Delta \ln q_{i,t} + (1 - \mathcal{L}_{i,t})\, g_{A_{i,t}},$$

where $q_{i,t}$ is the gross output (not value-added), and $\mathcal{L}_{i,t}$ is the Lerner index which is the ratio of price minus marginal cost to price.[34]

$$\mathcal{L}_{i,t} = 1 - \frac{1}{\mu_{i,t}}. \tag{30}$$

Hall (2018) assume that the Lerner index takes the following form:

$$\mathcal{L}_{i,t} = \phi_i + \psi_i \times t. \tag{31}$$

By using an IV estimation, Hall (2018) identifies (ϕ_i, ψ_i) for each industry i.

We use the same IV approach and take the same instruments as ones Hall (2018) uses. We estimate (ϕ_i, ψ_i) for sectors which produce equipment goods. We run the original regression by Hall (2018) and in addition, use the following specification to investigate whether the market power has increased after 2005:

$$\mathcal{L}_{i,t} = \phi_i + \psi_i \times t \times 1_{\{t \geq 2005\}}. \tag{32}$$

The market power rises if the Lerner index raise. This is because the Lerner index is an increasing function of the markup, (30). So, if we find a positive time trend under specification (31) or (32), then our interpretation that the relative price reflect the relative technological difference needs to be scrutinized.

The main estimation results are reported in Table 2. The regression results based on the original specification (31) are reported in column (1), (3), (5), and (7). The regression results based on specification (32) are reported in column (2), (4), (6), and (8). The results in Table 2 show that the Lerner indexes for these sectors do not have significant time trends in all the specifications. So, while other sectors have significant time trends (see Table 4 in Hall (2018)), the sectors which produce equipment do not. So we could not find evidence that these sectors have increased their market power. [35]

[34] For a discussion of the econometrics benefits of estimating the Lerner index, not markup, see Hall (2018).

[35] There is a subtle related issue. The results reported in Table 2 show that the computer and transportation sector have (time-invariant) market power. In such a case, the Solow residual does not correspond to their TFP. The corrected TFP for sector i is $g_{A_{i,t}} = (\Delta \ln s_{i,t} - \mathcal{L}_i \Delta \ln q_{i,t}) / (1 - \mathcal{L}_i)$. For the computer and transportation sector, we obtain their associated corrected TFP sequence based on their point estimate of the Lerner index, $\mathcal{L}_i$. We confirm that the average growth rate for these sectors before 2005 is slower than one after 2005.

Table 2: Lerner Index

	Computer		Electrical Equipment		Machinery		Transportation	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept ϕ_i	0.703	0.672	-0.139	-0.242	0.174	0.064	0.323	0.344
	(0.146)	(0.096)	(0.204)	(0.171)	(0.090)	(0.123)	(0.081)	(0.123)
Linear Trend ψ_i	0.004		0.021		0.010		0.009	
From 1988 - 2015	(0.022)		(0.028)		(0.012)		(0.010)	
Liner Trend ψ_i		0.013		0.051		0.028		0.004
From 2005 - 2015		(0.054)		(0.051)		(0.017)		(0.018)

NOTEs: Computer stands for "Computer and Electronic Products (NAICS 334)," Electrical Equipment stands for "Electrical Equipment, Appliances, and Components (NAICS 335)," Machinery stands for "Machinery (NAICS 333)," and Transportation stands for "Transportation Equipment (NAICS 336)." The numbers in parentheses represent the standard errors, which are calculated using the bootstrap method used in Hall (2018).

Although our results are based on industry-level data, most recent studies on markup have been based on firm-level data. Therefore, even if an increase in markup is not detected at the industry level, it may have increased at the firm level. De Loecker and Eeckhout (2021) extends the analysis of De Loecker et al. (2020) by estimating the markup for various countries. While the aggregate markup has gone up for some countries, no all the developed countries have experienced that. According to their results (Figure A.1 and Figure A.3), the aggregate markup by Germany and Japan have seldom increased after 2000. Given the fact that the stagnation of ALP growth is observed across the developed countries, the findings of De Loecker and Eeckhout (2021) imply that the rise of market power might be the unique factor for the ALP stagnation.

6.4 Rental Rates Estimated by JIP and BLS

To quantify technology stagnation, our analysis requires information on the share in rental costs. As mentioned above, many countries do not publish the information needed to compute the shares in rental costs, θ_a. Therefore, we used the properties of the BGP to indirectly determine the capital cost shares. We discuss the validity of this approach in this subsection.

Japan (JIP) and the US (BLS) publish the information about rental costs so that we can directly compute the shares in capital rental costs. We compare these estimated shares in rental costs with ones we have estimated using the properties of BGP. We summarize our findings in Table 3. The column named JIP and US shows the shares in rental costs estimated by JIP and BLS respectively. The columns named BGP shows our estimates of the shares in the rental costs.

The shares in the rental costs estimated by using a property of the BGP and those estimated by JIP are arguably similar. On the other hand, the rental cost shares of the US by our method differs slightly different from ones obtained in BLS.

To explore the implication of the differences on the growth, we re-conduct the quantification exercise in Section 5 for Japan and the US by using the rental rates estimated by JIP and BLS. To do so, we begin by recomputing the growth rates of the sectoral TFP $\left\{\tilde{g}_{A_{n,t}}\right\}_{n\in\mathcal{N}}$ for Japan and the US given the rental cost shares by JIP and BEA-BLS. Then we redo the quantification exercise in Section 5 by using $\left\{\tilde{g}_{A_{n,t}}\right\}_{n\in\mathcal{N}}$.

Recall that under our benchmark rental rate shares, the technological stagnation lowers the ALP growth rate by 0.32% for Japan and 0.83% for the US (See Table 1). If we quantify the technological stagnation by using the rental cost shares by JIP and BLS, then the stagnation lowers the ALP growth rate by 0.34% for Japan and 1.03% for the US. For both countries, the total effects from the technological stagnation are larger under the rental cost shares, but the magnitudes are quite similar. So we conclude that our quantification is robust even if we use the shares in the rental costs estimated by JIP and BLS.

Table 3: Rental Rates

Japan (JIP)	1995-2005		2005-2017		1995-2017	
Asset Title	Data	BGP	Data	BGP	Data	BGP
Communication Equipment	2.7%	3.2%	2.8%	2.8%	2.7%	3%
Computing Equipment	6.1%	5.8%	3.7%	3.7%	4.8%	4.8%
Transport Equipment	5.1%	5.6%	5.5%	6.3%	5.2%	5.9%
Other Equipment	24%	25%	24%	26%	23%	25%
Structure	46%	41%	42%	35%	44%	38%
Research & Development	10%	13%	14%	16%	12%	15%
Computer Software & Databases	5.4%	6.7%	8.2%	9.4%	6.7%	8%
Cultivated Assets	0.14%	0.16%	0.12%	0.17%	0.13%	0.16%
Other IPP Assets	0.57%	0%	0.53%	0.012%	0.55%	0%
US (BLS)	1995-2005		2005-2017		1995-2017	
Asset Title	Data	BGP	Data	BGP	Data	BGP
Communication Equipment	5.4%	3%	4.4%	2.5%	4.8%	2.9%
Computing Equipment	5.2%	2.3%	3.5%	1.8%	4.2%	2.1%
Transport Equipment	11%	5.7%	9.3%	6.9%	10%	6.1%
Other Equipment	26%	20%	22%	19%	24%	20%
Structure	29%	48%	35%	47%	32%	48%
Research & Development	12%	12%	13%	13%	13%	12%
Computer Software & Databases	7.5%	6.9%	9.3%	7.5%	8.6%	7.1%
Cultivated Assets	0%	0%	0%	0%	0%	0%
Other IPP Assets	4.1%	2.4%	3.6%	2.1%	3.8%	2.3%

Notes: (1) The data needed to create this table can be downloaded from the following two links: `https://www.rieti.go.jp/en/database/JIP2018/data/jip2018_2.xlsx` for Japan; and `https://www.bls.gov/mfp/special_requests/rental_price.xlsx` for the US. In the excel file for Japan, the sheet named as 'Capital cost by asset' provides the rental cost, $r_{a,t}K_{a,t-1}$ for each asset class and date. We compute the US rental cost, $r_{a,t}K_{a,t-1}$ by multiplying 'Actual rental price' by 'Productive Stock.' Our mapping from 'Asset Title' to the KLEMS classification is downloaded from this url, `https://www.dropbox.com/s/b54plzna6ceatw9/sector_classification.xlsx?dl=0`. (2) The column named "Data" represents the rental cost shares Japan and the US, and the column named "BGP" represents the rental cost shares we estimated using the properties of BGP. The average of the rental cost shares over different periods is shown.

7 What Drives Global Technological Stagnation After All?

Our empirical analysis has shown that the measured investment-specific technological stagnation can explain the stagnation among the developed countries. In this subsection, we discuss what sort of model is consistent with our empirical analysis. We emphasize three points from our empirical analysis: the measured technological stagnation is an global phenomenon; the stagnation of economic growth is the stagnation of technological progress and; the measured TFP growth rates slow down, not the TFP levels.

The results of our analysis indicate that the stagnation of investment-specific TFP is an global phenomenon among the developed countries. For this reason, it can be argued that the technological stagnation was not caused by factors specific to each country, nor was it caused by domestic factors. Many papers on secular stagnation seek to attribute the cause to domestic factors, and our study warns against such an approach. For example, Aoki et al. (2017) attempt to explain Japan's secular stagnation using a multiple equilibrium model. This theoretical model, although very interesting in itself, is incompatible with the results of our empirical analysis.

Our analysis also shows that the factor of technological stagnation should be one that leads to an increase in the relative price of investment goods. More specifically, the technological stagnation in equipment is the main driver of the technological stagnation of overall investment goods. Therefore, our analysis also raises alarm bells about several models. For example, many models that attribute stagnation purely to the demand side are incompatible with our empirical results. Many recent models (e.g. Eggertsson et al. (2019b)) deal with both demand-side problems (e.g. ZLB) and supply-side problems (technological stagnation) simultaneously. However, our analysis shows that supply-side problems alone can explain the bulk of stagnation in many countries, especially the US.

Our analysis also shows that the *growth* rate of technological progress is in stagnation. Therefore, models that lead to a stagnation in the *level* of technology cannot be used to derive our results. For example, some models of misallocation (e.g. Hsieh and Klenow (2012)) and some financial friction models (e.g. Buera and Moll (2015)) are known to lead to a decline in the *level* of measured TFP, but they alone cannot explain the stagnation in *growth* of the economy.

A model in which financial constraints have a long-run impact on growth rates appears to be consistent with our empirical results. However, given that technological stagnation has occurred mainly in equipment, it remains an open question why financial constraints have had a stronger impact on equipment-producing industries, not all the industries. It is valuable to analyze and quantify these differential sectoral effects using micro data, and further research is expected in the future.

On the other hand, our empirical work is consistent with models that would suggest technological stagnation. For example, a recent study by Bloom et al. (2020) analyzes a variety of industries and finds that it is becoming more difficult to innovate new technologies. Also, Gordon argues that the progress of technological innovation is slowing in general.

8 Conclusion

This paper explores the factors behind the stagnation in economic growth that is common among developed countries. We focused on the fact that the decline in the relative price of equipment goods has slowed globally, and interpreted the stagnation in the decline in relative prices as a technological stagnation. We build a simple growth model and quantify the effect of the technological stagnation on the economic growth. We find that this technological stagnation in the equipment sectors alone can explain the ALP stagnation across the developed countries, especially the US.

Because our analysis primarily used macroeconomic data, we cannot explore the true causes of technological stagnation. In order make a progress on this issue, it might be useful to apply the methods developed by empirical industrial organization literature to the equipment sectors and examine the evolution of technology, demand elasticity and market power of the sector. Such an analysis could be valuable and remain as a future research.

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A Exception Handling

We make the following changes to our data.

- The investment prices of equipment for the UK in the KLEMS dataset move very wild and differently from ones in the national account downloaded from OECDStat before 1997. So, we disregard all the observations before 1997 and only use the data about the investment price growth rates after 1998 for UK. These data can be downloaded from the following url. https://www.dropbox.com/sh/s6rwadj1bf460mv/AAD8ntX8Ij8KydtUsuvvlrPMa?dl=0 In the pdf file, "investment_price_comparison.pdf" in the folder, we depict the investment prices reported by the KLEMS dataset and the national account. https://www.dropbox.com/sh/s6rwadj1bf460mv/AAD8ntX8Ij8KydtUsuvvlrPMa?dl=0

- There are several minor changes made for Japan.
  - The data for Japan in the EU-KLEMS website is composed by Japan Industry Productivity and only has observations up to 2015. There is a new version of the dataset for Japan (JIP 2021), which has observations up to 2018. We supplement our dataset with JIP 2021 and the latest Japanese national account. JIP 2021 can be downloaded from this URL.[36]
  - In JIP 2021, there are no information about nominal capital stock by asset class. So, we download the nominal capital stocks from JSNA. JSNA does not report nominal capital stock for computer hardware (IT) and communication technology equipment (CT) separately. So, we compute the capital stock for them as follows. Let $K_{n,a,t}$ denote the nominal capital stock of $a \in \{\text{IT}, \text{CT}\}$. We measure $K_{n,a,t}$ as follows: for $a \in \{\text{IT}, \text{CT}\}$,

$$K_{n,a,t} = \frac{K_{a,t}^{\text{JIP}} p_{a,t}}{\sum_{a \in \{\text{IT},\text{CT}\}} K_{a,t}^{\text{JIP}} p_{a,t}} K_{n,\text{ICT},t}^{\text{JSNA}},$$

    where $K_{a,t}^{\text{JIP}}$ is the capital stock of a, $p_{a,t}$ is the associated investment deflator, and $K_{n,\text{ICT},t}^{\text{JSNA}}$ is the nominal capital stock of IT and CT. The other capital stocks for $a \in \mathcal{I} \setminus \{\text{IT}, \text{CT}\}$ are taken from JSNA.
  - We take the investment price sequence for other buildings and structures from JSNA. This is because JIP 2021 changes the method of deflation for this class of GFCF. Except for this asset class, the deflators for the investment goods used in JIP 2021 are almost identical to ones in JSNA.

[36] Unfortunately, the latest data is only available in Japanese. According to the JIP, they will publish the English version soon.

- Use Integrated BLS-BEA Industry-Level Production Account for g_Y, g_L, and α since the corresponding KLEMS data downloaded from EU-KLEMS website has a shorter sample period.

B Proofs

B.1 Lemma For Proposition 1

We use the following lemma to prove Proposition 1.

Lemma 1. *Along the BGP, the following hold:*

1. *the sectoral capital labor ratio of asset class a, is equal to the corresponding aggregate capital labor ratio,*
$$\frac{K_{a,t}}{L_t} = \frac{K_{a,n,t}}{L_{n,t}}; \tag{33}$$

2. *the sectoral capital service labor ratio is equal to the aggregate capital service labor ratio*
$$k_t = \frac{\prod_{a\in\mathcal{I}} K_{a,t}^{\theta_a}}{L_t} = \frac{\prod_{a\in\mathcal{I}} K_{a,n,t}^{\theta_a}}{L_{n,t}}; \tag{34}$$

3. *the rental rate of asset class a divided by the price of good a, $r_{a,t}/p_{a,t}$, is time-invariant.*

Proof. 1. Combining equation 7 with 8, we obtain

$$\frac{K_{a,n,t}}{L_{n,t}} = \frac{\alpha}{1-\alpha}\theta_a \frac{w_t}{r_{a,t}}.$$

Note the sectoral capital labor ratio of asset class a is independent of n. Recall that the market clearing of capital a is

$$K_{a,t} = \sum_{n\in\mathcal{N}} K_{a,n,t}. \tag{35}$$

So, the aggregate capital labor ratio of asset a is

$$\frac{K_{a,t}}{L_t} = \sum_{n\in\mathcal{N}} \frac{K_{a,n,t}}{L_t} = \sum_{n\in\mathcal{N}} \frac{L_{n,t}}{L_t} \times \frac{K_{a,n,t}}{L_{n,t}} = \frac{K_{a,n,t}}{L_{n,t}}.$$

For the first equality, we use equation (35). For the last equality, we use the fact that the sectoral labor ratio of asset class n is independent of $n \in \mathcal{N}$. We establish equation (33).

2. Equation (34) follows directly from equation (33).

$$\frac{\prod_{a\in\mathcal{I}} K_{a,n,t}^{\theta_a}}{L_{n,t}} = \prod_{a\in\mathcal{I}} \left(\frac{K_{a,n,t}}{L_{n,t}}\right)^{\theta_a} = \prod_{a\in\mathcal{I}} \left(\frac{K_{a,t}}{L_t}\right)^{\theta_a} = k_t.$$

To show the second equality, we use equation (33). Equation (34) is established.

3. The Euler equation along the BGP is

$$g_C = \ln \beta + \ln \left(\frac{r_{a,t+1}}{p_{a,t+1}} + (1 - \delta_a) \right) + g_{p_a}. \tag{36}$$

So, the real rental rate, $r_{a,t}/p_{a,t}$, is constant along the BGP. We complete the proof for Lemma B.1. □

B.2 Proof For Proposition 1

Proof. There are five properties in Proposition 1. We provide a proof for each.

1. Dividing equation 7 by $p_{a,t}$ and using equation (33), we obtain:

$$\frac{r_{a,t}}{p_{a,t}} = \alpha\theta_a \frac{p_{n,t}A_{n,t} \left(\prod_{a\in\mathcal{I}} K_{a,t}^{\theta_a}\right)^{\alpha} L_{n,t}^{1-\alpha}}{p_{a,t}K_{a,t}} = \alpha\theta_a A_{a,t} \frac{\left(\prod_{a\in\mathcal{I}} K_{a,t}^{\theta_a}\right)^{\alpha} L_t^{1-\alpha}}{K_{a,t}}. \tag{37}$$

For the second equality, we use equation (11). From Lemma 1, the rental rate, $r_{a,t}/p_{a,t}$, is time-invariant. So taking the logarithmic difference of equation (37), we obtain

$$0 = g_{A_a} + \alpha \sum_{a\in\mathcal{I}} \theta_a g_{K_a} + (1-\alpha)\, g_L - g_{K_a}. \tag{38}$$

Multiplying θ_a on both sides and taking the sum w.r.t. a, it is easy to show that the average growth rate of capital is given by

$$g_k = \sum_{a\in\mathcal{I}} \theta_a g_{K_a} - g_L = \frac{1}{1-\alpha} \sum_{a\in\mathcal{I}} \theta_a g_{A_a}, \tag{39}$$

which g_k is given by (23). Also the growth rate of the real capital stock of asset class a is

$$g_{K_a} = g_{A_a} + \alpha g_k + g_L. \tag{40}$$

2. We begin deriving the real rental rate for asset class $a \in \mathcal{I}$. Using equation (40), the sectoral nominal GDP growth rates are equalized and given by

$$\begin{aligned} \ln \frac{p_{n,t}Y_{n,t}}{p_{n,t-1}Y_{n,t-1}} &= \ln \frac{p_{n,t}A_{n,t} \left(\prod_{a\in\mathcal{I}} K_{a,n,t}^{\theta_a}\right)^{\alpha} L_{n,t}^{1-\alpha}}{p_{n,t-1}A_{n,t-1} \left(\prod_{a\in\mathcal{I}} K_{a,n,t-1}^{\theta_a}\right)^{\alpha} L_{n,t-1}^{1-\alpha}} \\ &= g_{A_C} + \alpha g_k + g_{L_n}. \end{aligned}$$

For the second equality we use equation (11). In order to obtain the third equality, we use equation (39). In particular we have:

$$g_C = \ln \frac{p_{C,t} Y_{C,t}}{p_{C,t-1} Y_{C,t-1}} = g_{A_C} + \alpha g_k + g_{L_C}. \tag{41}$$

Here we use the fact that the consumption good is taken as numeraire good. Substituting this expression into Euler equation (36), and using equation (11), for all a, the real rental price is time-invariant and given by

$$\frac{r_{a,t}}{p_{a,t}} = \beta^{-1} \exp\left(g_{A_a} + \alpha g_k + g_{L_C}\right) - (1-\delta_a). \tag{42}$$

Now we can derive the nominal value-added share of good $a \in \mathcal{I}$. The nominal share of $a \in \mathcal{I}$ is

$$\begin{aligned} s_a = \frac{p_{a,t} Y_{a,t}}{\sum_{n\in\mathcal{N}} p_{n,t} Y_{n,t}} &= \frac{p_{a,t}\left(K_{a,t+1} - (1-\delta_a) K_{a,t}\right)}{\sum_{n\in\mathcal{N}} p_{n,t} Y_{n,t}} \\ &= \frac{p_{a,t}\left(\frac{K_{a,t+1}}{K_{a,t}} - (1-\delta_a)\right) K_{a,t}}{\sum_{n\in\mathcal{N}} p_{n,t} Y_{n,t}} \\ &= \left(\exp\left(g_{K_a}\right) - (1-\delta_a)\right) \times \frac{p_{a,t}}{r_{a,t}} \frac{r_{a,t} K_{a,t}}{\sum_{n\in\mathcal{N}} p_{n,t} Y_{n,t}}. \end{aligned}$$

In order to obtain the third equality, we use the fact that the growth rate of the real capital stock of asset class a is given by equation (40). Because of the functional form assumptions on the production functions (see equation (3)), the capital service share for asset class a is $\alpha\theta_a$. So, the nominal share of good $a \in \mathcal{I}$ is further simplified to:

$$s_a = \left(\exp\left(g_{K_a}\right) - (1-\delta_a)\right) \alpha\theta_a \frac{p_{a,t}}{r_{a,t}} = \alpha \frac{\left(\exp\left(g_{A_a} + \alpha g_k + g_{L_C}\right) - (1-\delta_a)\right)}{\beta^{-1}\exp\left(g_{A_a} + \alpha g_k + g_{L_C}\right) - (1-\delta_a)} \theta_a. \tag{43}$$

In order to obtain the second equality, we use equation (42). From inequality (19), s_a is positive for all $a \in \mathcal{I}$. Also the total investment share, $\sum_{a\in\mathcal{I}} s_a$, satisfies

$$\sum_{a\in\mathcal{I}} s_a \leq \alpha \sum_{a\in\mathcal{I}} \theta_a = \alpha.$$

So, the consumption share is residually determined and strictly positive and less than 1:

$$s_C = \frac{p_{C,t} Y_{C,t}}{\sum_{n\in\mathcal{N}} p_{n,t} Y_{n,t}} = 1 - \sum_{n\in A} \frac{p_{n,t} Y_{n,t}}{\sum_{n\in\mathcal{N}} p_{n,t} Y_{n,t}} = 1 - \sum_{a\in\mathcal{I}} s_a.$$

It is easy to show that the value added shares coincide with the employment shares along the BGP.

To show, notice that

$$s_n = \frac{p_{n,t}Y_{n,t}}{\sum_{n\in\mathcal{N}} p_{n,t}Y_{n,t}} = \frac{A_{C,t}k_t^{\alpha}L_{n,t}}{\sum_{n\in\mathcal{N}} A_{C,t}k_t^{\alpha}L_{n,t}} = \frac{L_{n,t}}{\sum_{n\in\mathcal{N}} L_{n,t}}. \tag{44}$$

In order to obtain the second equality, we use equation (11) and (34). Equation (44) implies that the sectoral employment grows at the same rate as the aggregate labor input, g_L. So, the sectoral value added shares, $(s_n)_{n\in\mathcal{N}}$, are given

$$s_a = \alpha\frac{(\exp(g_{A_a} + \alpha g_k + g_L) - (1-\delta_a))}{\beta^{-1}\exp(g_{A_a} + \alpha g_k + g_L) - (1-\delta_a)}\theta_a \quad s_C = 1 - \sum_{a\in\mathcal{I}} s_a,$$

which is desired.

3. From the above argument, the nominal value added shares, $\{s_{n,t}\}_{n\in\mathcal{N}}$, stay constant. So, the real GDP growth rate along the BGP is

$$\begin{aligned} g_{V^*} &= \sum_{n\in\mathcal{N}} s_n \ln\frac{Y_{n,t}}{Y_{n,t-1}} \\ &= \sum_{n\in\mathcal{N}} s_n g_{A_n} + \alpha g_k + g_L. \end{aligned} \tag{45}$$

So the ALP growth rate is

$$g_{ALP} = \sum_{n\in\mathcal{N}} s_n g_{A_n} + \alpha g_K.$$

We establish equation (21).

4. Equation (22) follows immediately from equation (34) in Lemma (B.1).

We complete the proof for Proposition (1). □

B.3 Rental Rate Share Along Balanced Growth Path

Proposition 3. *(Proposition 3 in Gourio and Rognlie (2020)) Along the balanced growth path, the following equations hold for all $a\in\mathcal{I}$:*

$$\theta_a = \left(1-\alpha^{-1}s_I\right)s_a^I + \alpha^{-1}s_I s_a^K, \tag{46}$$

where s_a^K is the nominal share of capital stock evaluated by previous year replacement cost:

$$s_a^K = \frac{p_{a,t-1}K_{a,t}}{\sum_{a\in\mathcal{I}} p_{a,t-1}K_{a,t}}.$$

The share for asset class $a \in \mathcal{I}$ along the balanced growth path is

$$s_a^K = \frac{\frac{\theta_a \exp(g_{A_a})}{\beta^{-1}\exp(g_{A_a}+g_k+g_L)-(1-\delta_a)}}{\sum_{a\in\mathcal{I}} \frac{\theta_a \exp(g_{A_a})}{\beta^{-1}\exp(g_{A_a}+g_k+g_L)-(1-\delta_a)}}. \tag{47}$$

Proof. In order to prove this proposition we use the fact that the nominal capital stocks grow at the same constant rate. Recall the Euler equations by the representative household. Along the balanced growth path the consumption grows at a constant rate, and let r denote the net real interest rate of this economy. Then we have:

$$1 + r = \frac{r_{a,t} + (1-\delta_a)\, p_{a,t}}{p_{a,t-1}},$$

which is written as the so-called user cost formula:

$$r_{a,t} = (1+r)\, p_{a,t-1} - (1-\delta_a)\, p_{a,t}. \tag{48}$$

The following equation is obtained by rearranging the terms of the law of motion of capital for asset class a:

$$(1-\delta_a)\, K_{a,t} = K_{a,t+1} - I_{a,t}. \tag{49}$$

Multiplying $K_{a,t+1}$ on both sides of equation (48) and substituting equation (49) into equation (48), we obtain:

$$r_{a,t} K_{a,t} = \left[(1+r) - \frac{p_{a,t} K_{a,t+1}}{p_{a,t-1} K_{a,t}}\right] p_{a,t-1} K_{a,t} + p_{a,t} I_{a,t}. \tag{50}$$

Since the capital stock of asset class $a \in \mathcal{I}$ grows at rate g_{K_a} (40), the nominal capital stocks grow at the same constant rate along the BGP.

$$\begin{aligned}\ln \frac{p_{a,t} K_{a,t+1}}{p_{a,t-1} K_{a,t}} &= g_{K_a} + g_{p_a} = g_{A_a} + \frac{\alpha}{1-\alpha} \sum_{a\in\mathcal{I}} \theta_a g_{A_a} + g_L + g_{A_C} - g_{A_a} \\ &= g_{A_C} + g_k + g_L,\end{aligned}$$

which is common across $a \in \mathcal{I}$. To simplify our notation, let γ denote the net growth rate of the nominal capital stocks. It is easy to show that equation (50) is rewritten as follows:

$$r_{a,t} K_{a,t} = (r-\gamma)\, p_{a,t-1} K_{a,t} + p_{a,t} I_{a,t}. \tag{51}$$

Take the sum w.r.t. a of equation (51):

$$\sum_{a\in\mathcal{I}} r_{a,t}K_{a,t} = (r-\gamma)\sum_{a\in\mathcal{I}} p_{a,t-1}K_{a,t} + \sum_{a\in\mathcal{I}} p_{a,t}I_{a,t}. \tag{52}$$

Since the production functions are the Cobb-Douglass (see equation (3)),

$$\sum_{a\in\mathcal{I}} r_{a,t}K_{a,t} = \sum_{a\in\mathcal{I}}\sum_{n\in\mathcal{N}} \alpha\theta_a p_{n,t}Y_{n,t} = \alpha\sum_{n\in\mathcal{N}} p_{n,t}Y_{n,t}. \tag{53}$$

Divid equation (51) by (53), the following equation is derived:

$$\frac{r_{a,t}K_{a,t}}{\sum_{a\in\mathcal{I}} r_{a,t+1}K_{a,t}} = (r-\gamma)\frac{p_{a,t-1}K_{a,t}}{\alpha\sum_{n\in\mathcal{N}} p_{n,t}Y_{n,t}} + \frac{p_{a,t}I_{a,t}}{\alpha\sum_{n\in\mathcal{N}} p_{n,t}Y_{n,t}}. \tag{54}$$

Using equation (52) and definition of the nominal investment share, s_I, the first term in the RHS of equation (54) is rewritten as follows:

$$\begin{aligned}
(r-\gamma)\frac{p_{a,t-1}K_{a,t}}{\alpha\sum_{n\in\mathcal{N}} p_{n,t}Y_{n,t}} &= \frac{\sum_{a\in\mathcal{I}} p_{a,t}I_{a,t}}{\alpha\sum_{n\in\mathcal{N}} p_{n,t}Y_{n,t}}\frac{(r-\gamma)\sum_{a\in\mathcal{I}} p_{a,t-1}K_{a,t}}{\sum_{a\in\mathcal{I}} p_{a,t}I_{a,t}}\frac{p_{a,t-1}K_{a,t}}{\sum_{a\in\mathcal{I}} p_{a,t-1}K_{a,t}} \\
&= \alpha^{-1}s_I \times \frac{(r-\gamma)\sum_{a\in\mathcal{I}} p_{a,t-1}K_{a,t}}{\sum_{a\in\mathcal{I}} p_{a,t}I_{a,t}} \times s_a^K \\
&= \alpha^{-1}s_I \times \frac{\alpha\sum_{n\in\mathcal{N}} p_{n,t+1}Y_{n,t+1} - \sum_{a\in\mathcal{I}} p_{a,t+1}I_{a,t+1}}{\sum_{a\in\mathcal{I}} p_{a,t+1}I_{a,t+1}} \times s_a^K \\
&= \alpha^{-1}s_I \times \left(\alpha s_I^{-1} - 1\right) \times s_a^K \\
&= \left(1-\alpha^{-1}s_I\right) s_a^K.
\end{aligned}$$

In order to obtain the third equality, we use equation (53). It is easy to show that The second term of the RHS of equation (54) is equal to $\alpha^{-1}s_I s_a^I$. Therefore the rental rate θ_a satisfies

$$\theta_a = \frac{r_{a,t+1}K_{a,t}}{\sum_{a\in\mathcal{I}} r_{a,t+1}K_{a,t}} = \left(1-\alpha^{-1}s_I\right) s_a^K + \alpha^{-1}s_I s_a^I.$$

We establish equation (46).

Now we show that equation (47) holds for any $a \in \mathcal{I}$. Since

$$s_a^K = \frac{p_{a,t-1}K_{a,t}}{\sum_{a\in\mathcal{I}} p_{a,t-1}K_{a,t}} = \frac{\frac{p_{a,t-1}}{r_{a,t}} r_{a,t}K_{a,t}}{\sum_{a\in\mathcal{I}} \frac{p_{a,t-1}}{r_{a,t}} r_{a,t}K_{a,t}} = \frac{\frac{p_{a,t-1}}{r_{a,t}}\theta_a}{\sum_{a\in\mathcal{I}} \frac{p_{a,t-1}}{r_{a,t}}\theta_a}. \tag{55}$$

Recall the proof of Proposition (1) that the real rental rate of asset class a is given by equation

(42). Substituting equation (42) into equation (55), the nominal capital stock share is

$$s_a^K = \frac{\frac{\theta_a \exp(g_{A_a})}{\beta^{-1}\exp(g_{A_a}+g_k+g_L)-(1-\delta_a)}}{\sum_{a\in\mathcal{I}} \frac{\theta_a \exp(g_{A_a})}{\beta^{-1}\exp(g_{A_a}+g_k+g_L)-(1-\delta_a)}},$$

which is desired, (47). We established Proposition 3. □

B.4 Proof for Proposition 2

Proof. Using the fact that the total labor compensation is a fraction of the total nominal GDP,

$$(1-\alpha)\sum_{n\in\mathcal{N}} p_{n,t}Y_{n,t} = \sum_{n\in\mathcal{N}} w_{n,t}L_{n,t}$$
$$\alpha\sum_{n\in\mathcal{N}} p_{n,t}Y_{n,t} = \sum_{a\in\mathcal{I}} r_{a,t}K_{a,t},$$

we can rewrite the ratio ι_t as follows:

$$\iota_t = \frac{\sum_{a\in\mathcal{I}}(p_{a,t}I_{a,t} - \delta_a p_{a,t}K_{a,t})}{\sum_{a\in\mathcal{I}}(r_{a,t}K_{a,t} - \delta_a p_{a,t}K_{a,t})} = \sum_{a\in\mathcal{I}} \frac{\left(1-\delta_a\frac{p_{a,t}}{r_{a,t}}\right)\theta_a}{\sum_{b\in\mathcal{I}}\left(1-\delta_b\frac{p_{b,t}}{r_{b,t}}\right)\theta_b} \frac{\left(\frac{I_{a,t}}{K_{a,t}}-\delta_a\right)}{\left(\frac{r_{a,t}}{p_{a,t}}-\delta_a\right)} \tag{56}$$

Note that the real rental rate along the BGP is given by equation (42). By using equation (42), it is easy to show that

$$\left(1-\delta_b\frac{p_b}{r_b}\right) = 1 - \frac{\delta_b}{\beta^{-1}\exp(g_{K_b}) - (1-\delta_b)} = \frac{\beta^{-1}\exp(g_{K_b}) - 1}{\beta^{-1}\exp(g_{K_b}) - (1-\delta_b)},$$

where g_{K_b} is given by

$$g_{K_b} = g_{A_a} + \alpha g_k + g_{L_C}.$$

Also it is easy to show that

$$\frac{I_a}{K_a} - \delta_a = \exp(g_{K_a}) - 1.$$

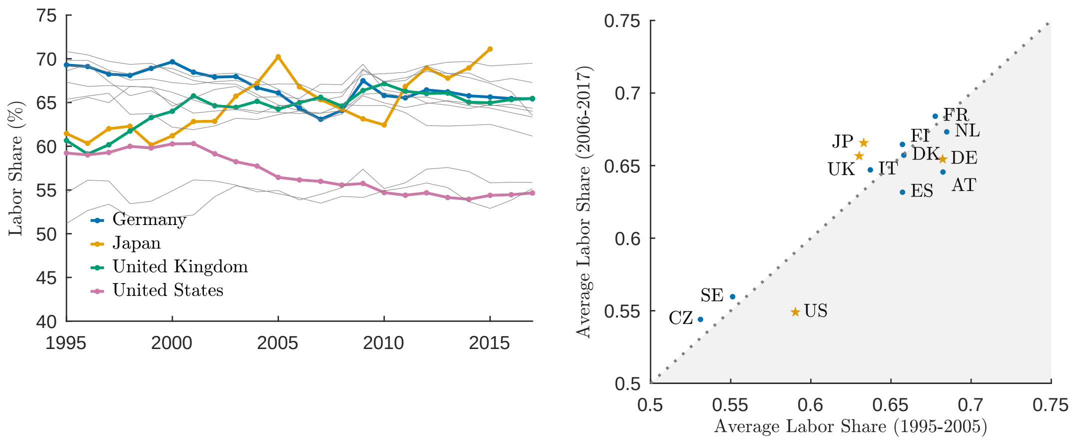
Substituting these equations into equation (56), we obtain the desired:

$$\iota_t = \sum_{a\in\mathcal{I}} \frac{\frac{\beta^{-1}\exp(g_{K_a})-1}{\beta^{-1}\exp(g_{K_a})-(1-\delta_a)}\theta_a}{\sum_{b\in\mathcal{I}}\frac{\beta^{-1}\exp(g_{K_b})-1}{\beta^{-1}\exp(g_{K_b})-(1-\delta_b)}\theta_b} \times \frac{\exp(g_{K_a})-1}{\beta^{-1}\exp(g_{K_a})-1}.$$

□

Figure 16: Labor Shares

(a) Time-Series of the Labor Shares (b) Changes of the Average Labor Shares



Notes: The labor shares are taken from the KLEMS dataset. The dotted lines represent the labor shares of Germany, Japan, the UK, and the US. The solid lines represent the shares for countries except these four countries. 16b plots the averages of the labor share until 2005 on the horizontal axis and the averages after 2005 on the vertical axis.

Online Appendix for
Tech-Driven Secular Stagnation: Cross-Country Evidence

Yuta Takahashi and Naoki Takayama

C Time-Varying Labor Shares

In the benchmark case, we assume that the labor shares, $1-\alpha$, are time-invariant and quantify the effect of the technology stagnation. In this section, we relax this assumption, and redo the quantification exercise again. It turns out that we get almost identical results even if we allow α to move over time. The rest of this section is organized as follows. We begin by plotting the time-series of the labor shares. Then we redo our quantification exercise in Section 5.

Figure 16a shows how the labor shares in the KLMES dataset move after 1995, and Figure 16b shows the average labor shares until 2005 and after 2005. These figures show that while the labor share of the US declined substantially, the decline of the labor share is not necessarily common across our sample. For example, Japan and the UK increased their labor shares slightly. Other counties like France and Italy have experienced no changes of the labor shares.

Now we allow the labor shares to move over time. Let $\tilde{\alpha}_t$ denote the date-t actual capital share reported in the KLMES dataset. We assume that until 2005, we take the value of α_t to be the average value of the actual capital share, $\tilde{\alpha}_t$, until 2005. After 2005, we assume the value of α_t

takes the average value after 2005.

$$\alpha_t = \begin{cases} \alpha_{\text{Pre}} & t \leq 2005 \\ \alpha_{\text{Post}} & t \geq 2006 \end{cases},$$

where

$$\alpha_{\text{Pre}} = \frac{\sum_{t=1995}^{2005} \tilde{\alpha}_t}{2005 - 1995 + 1},$$
$$\alpha_{\text{Post}} = \frac{\sum_{t=2006}^{2017} \tilde{\alpha}_t}{2017 - 2006 + 1}.$$

With this time-varying α, we redo our estimation of the sectoral TFP growth rates $\{g_{A_n}\}_{n\in\mathcal{N}}$. By using these new growth rates and the capital shares before 2006, α_{Pre}, we redo the quantification exercise in Section 5. The result is summarized in the following Table 4. We find that that the results are almost identical to the benchmark case even for the US. In the benchmark case, the technology stagnation of the US explains 59% of the decline of the ALP growth rate. If we allow the capital shares to move over time, the technology stagnation still explains 57% of the decline. From this exercise, we conclude that our argument is robust even if we allow the capital shares to move.

D Model of Mismeasurement

In this section, we examine how mismeasurement by the statistical agency affects the real GDP growth, our growth accounting procedure, and the main proposition. To do so, we assume that all the agents in this economy correctly see the prices and (therefore) quantities. But only the statistical agency cannot see the relevant prices and (therefore) quantities. Let $\tilde{p}_{n,t}$ denote the estimate of the relative price of good n at date t by the statistical agency. Suppose that the statistical agency mis-measures the relative growth rate of the prices: for all $n \in \mathcal{N}$,

$$g_{\tilde{p}_{n,t}} = g_{p_{n,t}} + \mu_{n,t}, \tag{57}$$

where $\mu_{n,t}$ governs the size of the mismeasurement of good $n \in \mathcal{N}$. Substituting equation (11) into equation (57), we obtain

$$g_{\tilde{p}_{n,t}} = -g_{A_{n,t}} + g_{A_{C,t}} + \mu_{n,t}.$$

This equation clarifies the role of the mismeasurement. When $\mu > 0$,then the statistical agency underestimates the impact of the technological improvement of good n relatively.

Table 4: Comparison Between Our Benchmark Case and the Case with Time-Varying α

	$\mathbf{d}g_{ALP}^{\text{Data}}$	$\mathbf{d}g_{ALP}^{\text{Tech}}$	
		Benchmark	Time-varying Capital Shares
Austria	-0.382%	0.012%	0%
Czech Republic	-1.56%	-1.217%	-1.227%
Denmark	-0.435%	-0.371%	-0.37%
Finland	-1.751%	-0.505%	-0.509%
France	-1.166%	-0.283%	-0.285%
Germany	-0.471%	-0.207%	-0.205%
Italy	-0.511%	-0.282%	-0.285%
Japan	-0.54%	-0.26%	-0.285%
Netherlands	-0.787%	-0.743%	-0.734%
Spain	0.577%	-0.505%	-0.493%
Sweden	-0.372%	-0.261%	-0.262%
United Kingdom	-1.214%	-0.754%	-0.78%
United States	-1.173%	-0.696%	-0.671%

Notes: The first column shows the difference in ALP growth rate until 2005 and after 2005. The second column shows the effect of technological stagnation on the ALP growth rate after 2005 under the assumption that the capital shares, α, are constant. The third column shows the effect of technology stagnation when the capital shares varies over time.

The measured growth rate of final good n is

$$\begin{aligned} g_{\tilde{Y}_{n,t}} &= \underbrace{g_{p_{n,t}} - g_{\tilde{p}_{n,t}}}_{\text{Mismeasurement}} + g_{Y_{n,t}} \\ &= g_{Y_{n,t}} - g_{\mu_{n,t}}. \end{aligned}$$

So, the measured real GDP growth rate, $g_{\tilde{V}_t^*}$, is given by

$$\begin{aligned} g_{\tilde{V}_t^*} &= \sum_{n \in \mathcal{N}} s_{n,t-1} g_{\tilde{Y}_{n,t}} \\ &= \sum_{n \in \mathcal{N}} s_{n,t-1} g_{Y_{n,t}} - \sum_{n \in \mathcal{N}} s_{n,t-1} g_{\mu_{n,t}} \\ &= g_{V_t^*} - \sum_{n \in \mathcal{N}} s_{n,t-1} g_{\mu_{n,t}} \end{aligned}$$

Note that if the mismeasurement and the nominal GDP share of good n is time-invariant, then the change of the measured real GDP growth rate corresponds to the true GDP growth rate:

$$\partial g_{\tilde{V}_t^*} = \partial g_{V_t^*}.$$

So, if the mismeasurement is time-invariant, then the mismeasurement alone cannot induce the ALP stagnation. Now consider the time-variant mismeasurement (but the constant nominal GDP shares). Then

$$\partial g_{\tilde{V}_t^*} = \partial g_{V_t^*} - \sum_{n \in \mathcal{N}} s_{n,t-1} \partial g_{\mu_{n,t}}.$$

Suppose that the true GDP growth rate does not change, $\partial g_{V_t^*} = 0$. Then the measured GDP stagnation only happens if

$$\sum_{n \in \mathcal{N}} s_{n,t-1} \partial g_{\mu_{n,t}} > 0.$$

Namely the amount of the mismeasurement needs to increase over time. Byrne et al. (2016) does not find evidence which supports this view.

Now suppose that the US quality adjustment is correct, and the quality adjustments by other countries are wrong. Then our dataset suggests that the mismeasurement by most countries is reduced after 2005. This is because the US computer price had declined more than the other countries until 2005, and after 2005, the relative computer prices across the countries were slower to drop than before. So, the difference between the growth rate of US relative price and the other countries shrinks. This is why the mismeasurement decreases for the other countries. Thus, if the US quality adjustment is correct, is correct, then the real GDP growth rate for the other countries would be higher, and the stagnation would not be caused by the mismeasurement.

Now we explore the implications of the mismeasurement for the growth accounting. Then the growth rate of the measured aggregated Solow residual, $g_{\tilde{A}_t}$, is

$$g_{\tilde{A}_t} = g_{V_t^*} - \alpha \sum_{a \in \mathcal{I}} \theta_a g_{\tilde{K}_{a,t}} - (1-\alpha)\, g_{L_t},$$

where $g_{\tilde{K}_{a,t}}$ is the growth rate of the measured capital stock of asset type a, $\tilde{K}_{a,t}$:

$$\tilde{K}_{a,t} \equiv \frac{p_{a,t} K_{a,t}}{\tilde{p}_{a,t}}.$$

So the growth rate of the measured capital stock of date t is

$$g_{\tilde{K}_{n,t}} = g_{p_{n,t}} - g_{\tilde{p}_{n,t}} + g_{K_{n,t}} = g_{K_{n,t}} - \mu_n.$$

For the second equality, we use equation (57). This equation is also intuitive. Since the statistical agency underestimates the capital-embodied technological change when $\mu > 0$, the real capital stocks are underestimated. We can back out the (wrong) sectoral TFP growth rates denoted by $\left\{g_{\tilde{A}_{n,t}}\right\}$ by solving the following equations:

$$g_{\tilde{A}_t} = \sum_{n\in\mathcal{N}} s_{n,t-1} g_{\tilde{A}_{n,t}}$$
$$g_{\tilde{p}_{n,t}} = -g_{\tilde{A}_n} + g_{\tilde{A}_C},$$

which imply that

$$g_{\tilde{A}_{n,t}} = g_{\tilde{A}_t} + \sum_{m\in\mathcal{N}} s_{m,t-1} g_{\tilde{p}_{m,t}} - g_{\tilde{p}_{n,t}}.$$

The difference between this TFP growth rate and the correct one is

$$\begin{aligned} g_{\tilde{A}_{n,t}} - g_{A_{n,t}} &= g_{\tilde{A}_t} - g_{A_t} + \sum_{m\in\mathcal{N}} s_{m,t-1}\left(g_{\tilde{p}_{m,t}} - g_{p_{m,t}}\right) - \left(g_{\tilde{p}_{n,t}} - g_{p_{n,t}}\right) \\ &= \alpha \sum_{n\in\mathcal{N}} s_{n,t-1}\mu_n - \mu_n. \end{aligned}$$

Assume that there are no mismeasurement related with non equipment goods. Then

$$g_{\tilde{A}_{n,t}} = g_{A_{n,t}} + \alpha \sum_{a\in\mathcal{E}} s_a \mu_a - \mu_n 1_{\{n\in\mathcal{E}\}}.$$

Then taking the total derivative, we obtain

$$\mathbf{d} g_{\tilde{A}_n} = \mathbf{d} g_{A_n} + \alpha \sum_{a\in\mathcal{E}} s_a \mathbf{d}\mu_a - \mathbf{d}\mu_n 1_{\{n\in\mathcal{E}\}}.$$

Thus if the amount of mismeasurement increase after 2005, then the sectoral TFP growth rate will be lower than the true TFP growth rate. Again, the literature does not find any evidence supporting this claim (Byrne et al. (2016)). If the amount of mismeasurement decreases then the true sectoral TFPs decline more.

E Quality Adjustment

In this section, we explain how the KLEMS dataset constructs its quality-adjusted labor service index. We begin by introducing the methodology of quality control by KLEMS. Then we proceed by providing an economic model rationalizing the methodology. This model makes it clear when the methodology for quality adjustment of the KLEMS dataset is effective and when it is not.

E.1 Methodology

The KLEMS dataset constructs the labor service index in two steps. First workers are grouped by type. Here, type refers to the characteristics of each worker. In the EU KLEMS, gender, age, and educational attainment are differentiated. In the Japanese KLEMS (JIP), on the top of these dimensions, employment status (e.g. self-employed, employee, part-time) is differentiated. After classifying workers, the growth rate of hours worked for each type is computed.

Second, the KLEMS dataset aggregates the growth rates of different types as follows, and construct the growth rate of the labor service index, L_t:

$$g_{L_t} = \sum_{i \in \Theta} \bar{\omega}_{i,t} g_{h_{i,t}}, \tag{58}$$

where Θ denote the set of all the types, $g_{h_{i,t}}$ is the growth rate of hours worked of type i at date t, and $\bar{\omega}_{i,t}$ is the average cost share of type i workers at date t and $t-1$:

$$\bar{\omega}_{i,t} = \frac{1}{2}\left[\frac{w_{i,t}h_{i,t}}{\sum_{i\in\Theta} w_{i,t}h_{i,t}} + \frac{w_{i,t-1}h_{i,t-1}}{\sum_{i\in\Theta} w_{i,t-1}h_{i,t-1}}\right].$$

Here $w_{i,t}$ is the nominal factor price of type i workers. So, the labor service index, L_t, differs from the total hours worked and take into account the fact that the factor prices are different. If all the workers the factor prices are the same, $w_{i,t} = w_{j,t}$ for all $i, j \in \Theta$, then g_{L_t} approximately corresponds to the growth rate of the total hours worked.[37]

E.2 Economic Model

Now we provide an economic model which rationalizes the labor service index, (58). There is a representative firm which produces a final good. The inputs for the production are capital, K_t, and various types of labor, $\mathbf{h}_t = (h_{t,i})_{i\in\Theta}$. Let F denote the production function. We assume that there is a constant-returns-to-scale, smooth, and quasi-concave function ϕ which aggregates the labor inputs as follows:

$$F(K_t, \mathbf{h}_t) = F(K_t, \phi(\mathbf{h}_t)).$$

The labor service L_t consists of hours worked by different types of workers:

$$L_t = \phi(\mathbf{h}_t), \tag{59}$$

[37] This correspondence is not exact due to Tornqvist approximation. If the nominal weight is $\left\{w_{i,t-1}h_{i,t-1}/\sum_{i\in\Theta} w_{i,t-1}h_{i,t-1}\right\}_{i\in\Theta}$, then the correspondence becomes exact.

Moreover, we assume that there is a frictionless labor market for each type $i \in \Theta$. So the firm solves the following minimization problem:

$$\min_{\{h_{i,t}\}_{i\in\Theta}} \quad \sum_{i\in\Theta} w_{i,t}h_{i,t} \tag{60}$$

$$s.t. \quad \phi\left((h_{i,t})_{i\in\Theta}\right) \geq L_t. \tag{61}$$

The key assumptions are: there is an aggregating CRS function g and; the labor input markets are perfectly competitive. Under the two assumptions, we show that the growth rate of L_t obtained under the firm's optimality condition corresponds to equation (58).

We begin by applying the Euler theorem to equation 59 to obtain

$$L_t = \sum_{i\in\Theta} \phi_i\left(\mathbf{h}_t\right) h_{i,t}. \tag{62}$$

Here, $\phi_i\left(\mathbf{h}_t\right)$ is the partial derivative of $\phi\left(\mathbf{h}_t\right)$ w.r.t. $h_{t.i}$. Then taking the logarithmic derivative of equation (62) with respect to t, we obtain:

$$g_{L_t} = \frac{1}{L_t}\left(\sum_{i\in\Theta} h_{i,t}\frac{\partial\phi_i\left(\mathbf{h}_t\right)}{\partial t} + \sum_{i\in\Theta}\phi_i\left(\mathbf{h}_t\right)h_{i,t}g_{h_{i,t}}\right).$$

Notice that the first summation term is zero:

$$\sum_{i\in\Theta} h_{i,t}\frac{\partial\phi_i\left(\mathbf{h}_t\right)}{\partial t} = \sum_{i\in\Theta}\sum_{j\in\Theta}\phi_{ij}\left(\mathbf{h}_t\right)\dot{h}_{j,t}h_{i,t} = \sum_{j\in\Theta}\left(\sum_{i\in\Theta}\phi_{ji}\left(\mathbf{h}_t\right)h_{i,t}\right)\dot{h}_{j,t} = 0.$$

The second equality comes from the fact that g is a smooth function, and the last equality comes from the fact that ϕ_j is homogeneous degree 0. So, the growth rate of the labor service index is written as follows:

$$g_{L_t} = \sum_{i\in\Theta}\frac{\phi_i\left(\mathbf{h}_t\right)h_{i,t}}{L_t}g_{h_{i,t}}. \tag{63}$$

Let λ_t denote the Lagrange multiplier for the constraint, (61). The FONCs for CMP (60) are

$$w_{i,t} = \lambda_t\phi_i\left(\mathbf{h}_t\right). \tag{64}$$

Multiplying $h_{i,t}$ on both sides and taking the sum w.r.t. $i \in \Theta$, we obtain:

$$\sum_{i\in\Theta} w_{i,t}h_{i,t} = \lambda_t\sum_{i\in\Theta}\phi_i\left(\mathbf{h}_t\right)h_{i,t} = \lambda_t L_t.$$

Combining this expression with equation (61), the elasticity of ϕ w.r.t. $h_{i,t}$ corresponds to the cost share of workers with type i.

$$\frac{\phi_i\left(\mathbf{h}_t\right) h_{i,t}}{L_{i,t}} = \frac{w_{i,t} h_{i,t}}{\sum_{i\in\Theta} w_{i,t} h_{i,t}}.$$

Therefore, the growth rate of the labor service, (63), is written as the weighted average of hours worked by different types of workers:

$$g_{L_t} = \sum_{i\in\Theta} \frac{w_{i,t} h_{i,t}}{\sum_{i\in\Theta} w_{i,t} h_{i,t}} g_{h_{i,t}}. \tag{65}$$

Notice that this labor service index is a generalization of the total hours worked. If workers are perfectly substitutable and $\phi\left(\mathbf{h}_t\right) = \sum_{i\in\Theta} h_{t,i}$, then equation (65) boils down to the growth rate of the total hours worked. To see this point , notice if all the workers are substitutable, then the firm only hire the workers whose wage is the lowest. In an interior equilibrium where $h_{i.t}$ is strictly positive, the wages should be equalized, $w_{i,t} = w_t$. Then equation (65) becomes

$$g_{L_t} = \frac{\sum_{i\in\Theta} \dot{h}_{i,t}}{\sum_{i\in\Theta} h_{i,t}},$$

which is the growth rate of the total hours worked.

If we apply Tornqvist approximation to equation (65), then we obtain equation (65).

$$g_{L_t} = \sum_{i\in\Theta} \bar{\omega}_{i,t} g_{h_{i,t}}. \tag{66}$$

In sum, we provide an economic model which justifies the labor service index constructed by the KLEMS dataset. The two key assumptions are: there is a CRS function ϕ which aggregates hours worked by different types of workers; and there is a frictionless labor market for each type $\theta \in \Theta$.

The labor service index, (66), is a better measure of labor input that generalizes total hours worked. Total hours worked is a correct measure of labor input only when different types of labor input are perfectly substitutable. However, such an assumption is unrealistic. Obviously the wages for workers are not equalized. The literature reports imperfect substitution between college and high school labor. For example, Katz and Murphy (1992) finds that an elasticity of substitution between college and high school labor is around 1.4, which is far from perfect substitute. The labor service index, (66), is the correct measure of labor input even if labor inputs are not perfectly substitutable under the key two assumptions.

F Model With Tax

We augment the model in Section 3 by incorporating various taxes. These taxes are imposed to the representative households, and do not affect the behaviors by the firms. We begin by introducing our model with taxes and then proceed by proving a version of Proposition 1 and 3.

F.1 Model Description

The government imposes the consumption, income, and capital taxes for the representative households. Let τ^C, τ^L, and τ^K denote the (time-invariant) consumption tax, labor income tax, and capital tax. The government does not issue any debt so that the budget balance holds for any $t \geq 0$.

The budget constraint at date t for the representative household is modified as follows:

$$p_{C,t}\left(1+\tau^C\right)C_t+\sum_{a\in\mathcal{I}}p_{a,t}K_{a,t+1} \leq w_t\left(1-\tau^L\right)L_t+\left(1-\tau^K\right)\sum_{a\in\mathcal{I}}\left(r_{a,t}-p_{a,t}\delta_a\right)K_{a,t}+\sum_{a\in\mathcal{I}}p_{a,t}K_{a,t}+T_t, \tag{67}$$

where T_t denotes the lump-sum transfer from the government. Given the flow budget constraints, the representative household maximizes its utility (1) subject to the constraints, (67). The FONCs are

$$\frac{1}{p_{C,t}C_t} = \beta\frac{1}{p_{C,t+1}C_{t+1}}\left(\frac{\left(1-\tau^K\right)\left(r_{a,t+1}-p_{a,t+1}\delta_a\right)+p_{a,t+1}}{p_{a,t}}\right)$$

for all $a \in \mathcal{I}$ and $t \geq 0$.

The flow budget constraint for the government is given by

$$T_t = p_{C,t}\tau^C C_t + w_t\tau^L L_t + \tau^K\sum_{a\in\mathcal{I}}\left(r_{a,t}-p_{a,t}\delta_a\right)K_{a,t}.$$

As mentioned above, the firms' maximization problems are the same as ones in Section (3). A competitive equilibrium is defined as usual.

F.2 Growth Rate Along Balanced Growth Path

Assume that the sectoral TFPs and labor grow at constant rates. We derive the growth rate of the real GDP and ALP along the balanced growth path.

Proposition 4. *Suppose that for all $a \in \mathcal{I}$,*

$$g_{A_a} + \frac{\alpha}{1-\alpha}\sum_{a\in\mathcal{I}}\theta_a g_{A_a} + g_L > \ln\left(1-\delta_a\right). \tag{68}$$

The real GDP growth rate along the BGP, g_{V^*}, is

$$g_{ALP} = \sum_{n\in\mathcal{N}} s_n g_{A_n} + \alpha g_k, \tag{69}$$

where s_n is the nominal value added share of sector n along the BGP and g_k is given by

$$g_k = \frac{1}{1-\alpha}\sum_{a\in\mathcal{I}} \theta_a g_{A_a}. \tag{70}$$

The nominal value added shares, $(s_n)_{n\in\mathcal{N}}$, along the BGP are given by

$$\begin{aligned} s_a &= \frac{\alpha\left(1-\tau^K\right)\left(\exp\left(g_{A_a}+\alpha g_k+g_L\right)-(1-\delta_a)\right)}{\beta^{-1}\exp\left(g_{A_a}+\alpha g_k+g_L\right)-\left(1-\delta_a+\tau^K\delta_a\right)}\theta_a \quad a\in\mathcal{I} \\ s_C &= 1-\sum_{a\in\mathcal{I}} s_a. \end{aligned}$$

Note that the consumption and labor income taxes do not affect the growth rate of the economy and the sectoral shares of the GDP. This invariance result comes from the fact that the representative agent supplies its labor inelastically.

Proof. The proof consists of the same two parts as in the proof for Proposition 3. In the first part, we derive the ALP growth rate, (21), along the BGP given the sectoral nominal value added shares. In the second part, we derive the sectoral value-added shares along the BGP.

(1) Along the BGP, the consumption expenditure grows at a constant rate. So, the Euler equation along the balanced growth path is reduced to:

$$\beta^{-1}\frac{C_{t+1}}{C_t} = \left(\left(1-\tau^K\right)\frac{r_{a,t+1}}{p_{a,t+1}} + 1 - \delta_a + \tau^K\delta_a\right)\frac{p_{a,t+1}}{p_{a,t}} \tag{71}$$

Since along the BGP, the price of asset class $a\in\mathcal{I}$ grows at a constant rate, the real rental rate is time-invariant along the BGP, $r_{a,t+1}/p_{a,t+1} = r_{a,t}/p_{a,t}$. The rest of the proof for step 1 is the same as one in Proposition 3. The growth rate of the real capital stocks, the real GDP, and the ALP are the same as ones in Proposition 3.

(2) Now we derive the nominal value-added shares, which differ from ones in Proposition 3.

Using equation (40), the nominal GDP growth rate of good n is

$$\begin{aligned}
\ln \frac{p_{n,t}Y_{n,t}}{p_{n,t-1}Y_{n,t-1}} &= \ln \frac{p_{n,t}A_{n,t}\left(\prod_{a\in\mathcal{I}} K_{a,n,t}^{\theta_a}\right)^{\alpha} L_{n,t}^{1-\alpha}}{p_{n,t-1}A_{n,t-1}\left(\prod_{a\in\mathcal{I}} K_{a,n,t-1}^{\theta_a}\right)^{\alpha} L_{n,t-1}^{1-\alpha}} \\
&= g_{A_C} + (1-\alpha)\, g_L + \alpha \sum_{a\in\mathcal{I}} \theta_a g_{K_a} \\
&= g_{A_C} + (1-\alpha)\, g_L + \alpha\,(g_k + g_L) \\
&= g_{A_C} + \alpha g_k + g_L.
\end{aligned}$$

In order to obtain the third equality, we use equation (23). The consumption good price is normalized to 1. So the consumption grows at the rate,

$$g_C = \ln \frac{p_{C,t}Y_{C,t}}{p_{C,t-1}Y_{C,t-1}} = g_{A_C} + \alpha g_k + g_L.$$

Substituting this expression into Euler equation (71), and using equation (11), for all a, the real rental price is time-invariant and given by

$$\frac{r_{a,t+1}}{p_{a,t+1}} = \frac{\beta^{-1}\exp\left(g_{A_a} + \alpha g_k + g_L\right) - \left(1 - \delta_a + \tau^K \delta_a\right)}{1-\tau^K}. \tag{72}$$

Now we can derive the nominal value-added share for investment good $a \in \mathcal{I}$. The nominal share for $a \in \mathcal{I}$ is

$$\begin{aligned}
s_a = \frac{p_{a,t}Y_{a,t}}{\sum_{n\in\mathcal{N}} p_{n,t}Y_{n,t}} &= \frac{p_{a,t}\left(K_{a,t+1} - (1-\delta_a)\,K_{a,t}\right)}{\sum_{n\in\mathcal{N}} p_{n,t}Y_{n,t}} \\
&= \frac{p_{a,t}\left(\frac{K_{a,t+1}}{K_{a,t}} - (1-\delta_a)\right)K_{a,t}}{\sum_{n\in\mathcal{N}} p_{n,t}Y_{n,t}} \\
&= \left(\exp\left(g_{A_a} + \alpha g_k + g_L\right) - (1-\delta_a)\right) \times \frac{p_{a,t}}{r_{a,t}} \frac{r_{a,t}K_{a,t}}{\sum_{n\in\mathcal{N}} p_{n,t}Y_{n,t}}.
\end{aligned}$$

In order to obtain the third equality, we use the fact that the growth rate of the real capital stock of asset class a is given by equation (40). Since the production functions are identical and Cobb-Douglas (see equation (3)), the capital service share for asset class a is $\alpha\theta_a$. So, the nominal share is further simplified to:

$$s_a = \left(\bar{g}_{K_a} - (1-\delta_a)\right)\alpha\theta_a \frac{p_{a,t}}{r_{a,t}} = \alpha\left(1-\tau^K\right)\frac{\left(\exp\left(g_{A_a} + \alpha g_k + g_L\right) - (1-\delta_a)\right)}{\beta^{-1}\exp\left(g_{A_a} + \alpha g_k + g_L\right) - \left(1-\delta_a+\tau^K\delta_a\right)}\theta_a. \tag{73}$$

The consumption share is residually determined:

$$s_C = \frac{p_{C,t} Y_{C,t}}{\sum_{n \in \mathcal{N}} p_{n,t} Y_{n,t}} = 1 - \sum_{n \in A} \frac{p_{n,t} Y_{n,t}}{\sum_{n \in \mathcal{N}} p_{n,t} Y_{n,t}} = 1 - \sum_{a \in \mathcal{I}} s_a,$$

which completes the proof. □

F.3 Rental Rate Share Along Balanced Growth Path

Proposition 5. *Along the balanced growth path, the following equations hold for all $a \in \mathcal{I}$:*

$$\theta_a = \left(1 - \tau^K - \alpha^{-1} s_I + \alpha^{-1} \tau^K d\right) s_a^K + \left(\alpha^{-1} s_I\right) s_a^I + \left(-\alpha^{-1} \tau^K d\right) s_a^\delta, \tag{74}$$

where s_a^K is the nominal share of capital stock evaluated by previous year replacement cost, d is the ratio of deprecation to the GDP, and s_a^δ is the share of depreciation of asset a:

$$s_a^K = \frac{p_{a,t-1} K_{a,t}}{\sum_{a \in \mathcal{I}} p_{a,t-1} K_{a,t}} \quad d = \frac{\sum_{a \in \mathcal{I}} p_{a,t} \delta_a K_{a,t}}{\sum_{n \in \mathcal{N}} p_{n,t} Y_{n,t}}, \quad s_a^\delta = \frac{p_{a,t} \delta_a K_{a,t}}{\sum_{a \in \mathcal{I}} p_{a,t} \delta_a K_{a,t}}$$

The replacement cost share for asset class $a \in \mathcal{I}$, the depreciation share d, and the share of depreciation for asset class $a \in \mathcal{I}$ along the balanced growth path are

$$s_a^K = \frac{\frac{\theta_a \exp(g_{A_a})}{\beta^{-1} \exp(g_{A_a} + g_k + g_L) - (1 - \delta_a)}}{\sum_{a \in \mathcal{I}} \frac{\theta_a \exp(g_{A_a})}{\beta^{-1} \exp(g_{A_a} + g_k + g_L) - (1 - \delta_a)}}, \tag{75}$$

$$d = \alpha \sum_{a \in \mathcal{I}} \frac{\theta_a \delta_a}{\beta^{-1} \exp\left(g_{A_a} + g_k + g_L\right) - (1 - \delta_a)}, \tag{76}$$

$$s_a^\delta = \frac{\frac{\delta_a \theta_a}{\beta^{-1} \exp(g_{A_a} + g_k + g_L) - (1 - \delta_a)}}{\sum_{a \in \mathcal{I}} \frac{\delta_a \theta_a}{\beta^{-1} \exp(g_{A_a} + g_k + g_L) - (1 - \delta_a)}}. \tag{77}$$

Proof. In order to prove this proposition we use the fact that the nominal capital stocks grow at the same constant rate. Recall the Euler equations by the representative household. Along the balanced growth path the consumption grows at a constant rate, and let r denote the net real interest rate of this economy. Then we have:

$$1 + r = \frac{r_{a,t+1} + (1 - \delta_a) p_{a,t+1}}{p_{a,t}} - \tau^K \frac{(r_{a,t+1} - \delta_a p_{a,t+1})}{p_{a,t}},$$

which is written as the so-called user cost formula:

$$\left(1 - \tau^K\right) r_{a,t} = (1 + r) p_{a,t-1} - (1 - \delta_a) p_{a,t} - \tau^K \delta_a p_{a,t}. \tag{78}$$

The following equation is obtained by rearranging the terms of the law of motion of capital for asset class a:

$$(1-\delta_a) K_{a,t} = K_{a,t+1} - I_{a,t}. \tag{79}$$

Multiplying $K_{a,t+1}$ on both sides of equation (78) and substituting equation (79) into equation (78), we obtain:

$$\left(1-\tau^K\right) r_{a,t} K_{a,t} = \left[(1+r) - \frac{p_{a,t}K_{a,t+1}}{p_{a,t-1}K_{a,t}}\right] p_{a,t-1}K_{a,t} + p_{a,t}I_{a,t} - \tau^K p_{a,t}\delta_a K_{a,t}. \tag{80}$$

Since the capital stock of asset class $a \in \mathcal{I}$ grows at rate g_{K_a} (40), the nominal capital stocks grow at the same constant rate along the BGP.

$$\begin{aligned}\ln \frac{p_{a,t}K_{a,t+1}}{p_{a,t-1}K_{a,t}} &= g_{K_a} + g_{p_a} = g_{A_a} + \frac{\alpha}{1-\alpha}\sum_{a\in\mathcal{I}}\theta_a g_{A_a} + g_L + g_{A_C} - g_{A_a} \\ &= g_{A_C} + g_k + g_L,\end{aligned}$$

which is common across $a \in \mathcal{I}$. To simplify our notation, let γ denote the net growth rate of the nominal capital stocks. It is easy to show that equation (80) is rewritten as follows:

$$\left(1-\tau^K\right) r_{a,t} K_{a,t} = (r-\gamma)\, p_{a,t-1}K_{a,t} + p_{a,t}I_{a,t} - \tau^K p_{a,t}\delta_a K_{a,t}. \tag{81}$$

Take the sum w.r.t. a of equation (81):

$$\left(1-\tau^K\right) \sum_{a\in\mathcal{I}} r_{a,t} K_{a,t} = (r-\gamma) \sum_{a\in\mathcal{I}} p_{a,t-1}K_{a,t} + \sum_{a\in\mathcal{I}} p_{a,t}I_{a,t} - \tau^K p_{a,t}\delta_a K_{a,t}. \tag{82}$$

Since the production functions are the Cobb-Douglass (see equation (3)),

$$\sum_{a\in\mathcal{I}} r_{a,t}K_{a,t} = \sum_{a\in\mathcal{I}}\sum_{n\in\mathcal{N}} \alpha\theta_a p_{n,t}Y_{n,t} = \alpha \sum_{n\in\mathcal{N}} p_{n,t}Y_{n,t}. \tag{83}$$

Divid equation (81) by (83), the following equation is derived:

$$\left(1-\tau^K\right) \frac{r_{a,t}K_{a,t}}{\sum_{a\in\mathcal{I}} r_{a,t+1}K_{a,t}} = (r-\gamma)\frac{p_{a,t-1}K_{a,t}}{\alpha\sum_{n\in\mathcal{N}} p_{n,t}Y_{n,t}} + \frac{p_{a,t}I_{a,t}}{\alpha\sum_{n\in\mathcal{N}} p_{n,t}Y_{n,t}} - \tau^K \frac{p_{a,t}\delta_a K_{a,t}}{\alpha\sum_{n\in\mathcal{N}} p_{n,t}Y_{n,t}}. \tag{84}$$

Using equation (82) and definition of the nominal investment share, s_I, the first term in the RHS of equation (84) is rewritten as follows:

$$\begin{aligned}
(r-\gamma)\frac{p_{a,t-1}K_{a,t}}{\alpha\sum_{n\in\mathcal{N}}p_{n,t}Y_{n,t}} &= \frac{\sum_{a\in\mathcal{I}}p_{a,t}I_{a,t}}{\alpha\sum_{n\in\mathcal{N}}p_{n,t}Y_{n,t}}\frac{(r-\gamma)\sum_{a\in\mathcal{I}}p_{a,t-1}K_{a,t}}{\sum_{a\in\mathcal{I}}p_{a,t}I_{a,t}}\frac{p_{a,t-1}K_{a,t}}{\sum_{a\in\mathcal{I}}p_{a,t-1}K_{a,t}}\\
&= \alpha^{-1}s_I\times\frac{(r-\gamma)\sum_{a\in\mathcal{I}}p_{a,t-1}K_{a,t}}{\sum_{a\in\mathcal{I}}p_{a,t}I_{a,t}}\times s_a^K\\
&= \alpha^{-1}s_I\times\frac{\left(1-\tau^K\right)\alpha\sum_{n\in\mathcal{N}}p_{n,t}Y_{n,t}-\sum_{a\in\mathcal{I}}p_{a,t}I_{a,t}+\tau^K\sum_{a\in\mathcal{I}}p_{a,t}\delta_aK_{a,t}}{\sum_{a\in\mathcal{I}}p_{a,t+1}I_{a,t+1}}\times s_a^K\\
&= \alpha^{-1}s_I\times\left(\left(1-\tau^K\right)\alpha s_I^{-1}-1+\tau^K\frac{d}{s_I}\right)\times s_a^K\\
&= \left(1-\tau^K-\alpha^{-1}s_I+\alpha^{-1}\tau^Kd\right)\times s_a^K.
\end{aligned}$$

In order to obtain the third equality, we use equation (83). It is easy to show that the second term of the RHS of equation (84) is equal to $\alpha^{-1}s_Is_a^I$. The last term in equation (84) is

$$\begin{aligned}
\frac{p_{a,t}\delta_aK_{a,t}}{\alpha\sum_{n\in\mathcal{N}}p_{n,t}Y_{n,t}} &= \frac{\sum_{a\in\mathcal{I}}p_{a,t}\delta_aK_{a,t}}{\alpha\sum_{n\in\mathcal{N}}p_{n,t}Y_{n,t}}\frac{p_{a,t}\delta_aK_{a,t}}{\sum_{a\in\mathcal{I}}p_{a,t}\delta_aK_{a,t}}\\
&= \alpha^{-1}d\times d_a.
\end{aligned}$$

Therefore the rental rate θ_a satisfies

$$\theta_a=\frac{r_{a,t+1}K_{a,t}}{\sum_{a\in\mathcal{I}}r_{a,t+1}K_{a,t}}=\left(1-\tau^K-\alpha^{-1}s_I+\alpha^{-1}\tau^Kd\right)s_a^K+\left(\alpha^{-1}s_I\right)s_a^I+\left(-\alpha^{-1}\tau^Kd\right)s_a^\delta.$$

We establish equation (74).

Now we show that equation (75) holds for any $a\in\mathcal{I}$. Since

$$s_a^K=\frac{p_{a,t-1}K_{a,t}}{\sum_{a\in\mathcal{I}}p_{a,t-1}K_{a,t}}=\frac{\frac{p_{a,t-1}}{r_{a,t}}r_{a,t}K_{a,t}}{\sum_{a\in\mathcal{I}}\frac{p_{a,t-1}}{r_{a,t}}r_{a,t}K_{a,t}}=\frac{\frac{p_{a,t-1}}{r_{a,t}}\theta_a}{\sum_{a\in\mathcal{I}}\frac{p_{a,t-1}}{r_{a,t}}\theta_a}. \tag{85}$$

Recall the proof of Proposition (1) that the real rental rate of asset class a is given by equation (42). Substituting equation (42) into equation (85), the nominal capital stock share is

$$s_a^K=\frac{\frac{\theta_a\exp(g_{A_a})}{\beta^{-1}\exp(g_{A_a}+g_k+g_L)-(1-\delta_a)}}{\sum_{a\in\mathcal{I}}\frac{\theta_a\exp(g_{A_a})}{\beta^{-1}\exp(g_{A_a}+g_k+g_L)-(1-\delta_a)}},$$

which is desired, (75).

Now we derive the expression for the depreciation d along the balanced growth path.

$$d=\frac{\sum_{a\in\mathcal{I}}p_{a,t}\delta_aK_{a,t}}{\sum_{n\in\mathcal{N}}p_{n,t}Y_{n,t}}=\frac{\sum_{a\in\mathcal{I}}\delta_a\frac{p_{a,t}}{r_{a,t}}r_{a,t}K_{a,t}}{\sum_{n\in\mathcal{N}}p_{n,t}Y_{n,t}}.$$

From the functional form assumption about the production function, (3), we have

$$\alpha\theta_a p_{n,t} Y_{n,t} = r_{a,t} K_{a,n,t}. \tag{86}$$

Summing over $n \in \mathcal{N}$ and $a \in \mathcal{I}$, and using the capital market clearing condition, equation (86) boils down to:

$$\alpha\theta_a \sum_{n\in\mathcal{N}} p_{n,t} Y_{n,t} = r_{a,t} K_{a,t}.$$

So, the aggregate depreciation rate is

$$\begin{aligned} d &= \frac{\sum_{a\in\mathcal{I}} \delta_a \frac{p_{a,t}}{r_{a,t}} r_{a,t} K_{a,t}}{\sum_{n\in\mathcal{N}} p_{n,t} Y_{n,t}} \\ &= \alpha \sum_{a\in\mathcal{I}} \theta_a \delta_a \frac{p_{a,t}}{r_{a,t}} \\ &= \alpha \sum_{a\in\mathcal{I}} \frac{\theta_a \delta_a}{\beta^{-1}\exp\left(g_{A_a} + g_k + g_L\right) - (1-\delta_a)}. \end{aligned}$$

In order to obtain the last equality, we use equation (42).

We now proceed by showing the depreciation share, s_a^δ :

$$s_a^\delta = \frac{p_{a,t}\delta_a K_{a,t}}{\sum_{a\in\mathcal{I}} p_{a,t}\delta_a K_{a,t}}.$$

It is easy to show that

$$s_a^\delta = \frac{\delta_a \frac{p_{a,t}}{r_{a,t}} r_{a,t} K_{a,t}}{\sum_{a\in\mathcal{I}} \delta_a \frac{p_{a,t}}{r_{a,t}} r_{a,t} K_{a,t}} = \frac{\frac{p_{a,t}}{r_{a,t}}\delta_a\theta_a}{\sum_{a\in\mathcal{I}} \frac{p_{a,t}}{r_{a,t}}\delta_a\theta_a}.$$

In order to obtain the second equality, we use the assumption that $r_{a,t}K_{a,t}$ is proportional to the output. Substituting equation (42) into above equation, we obtain

$$s_a^\delta = \frac{\frac{\delta_a\theta_a}{\beta^{-1}\exp(g_{A_a}+g_k+g_L)-(1-\delta_a)}}{\sum_{a\in\mathcal{I}} \frac{\delta_a\theta_a}{\beta^{-1}\exp(g_{A_a}+g_k+g_L)-(1-\delta_a)}},$$

which is desired. Now we establish Proposition 5. □

G Additional Figures

Figure 17: Labor Input

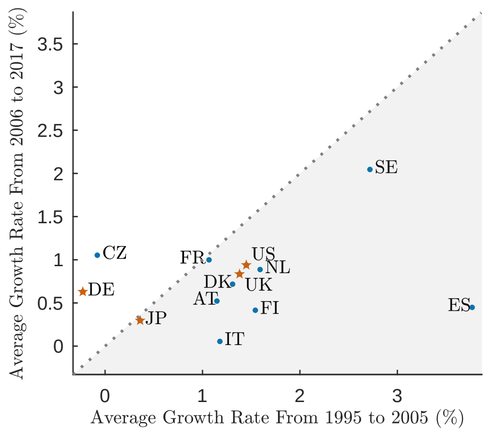
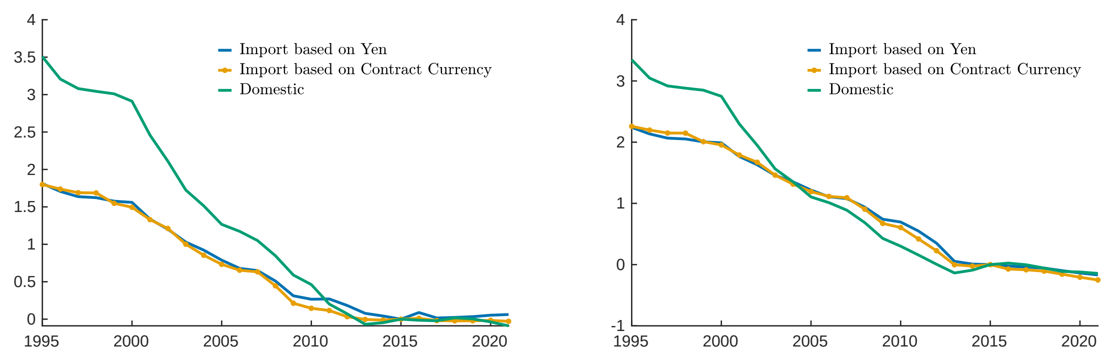


Figure 18: Evidence From Japanese PPI

(a) Desktop PC (b) Notebook PC



H Another Definition of Real GDP

Greenwood et al. (1997) and Herrendorf et al. (2014) define the real GDP as follows:

$$V_t^C = \frac{\sum_{n\in\mathcal{N}} p_{n,t} Y_{n,t}}{p_{C,t}}. \tag{87}$$

By using equation (11) and equation (34) in Lemma B.1, we can show that the real GDP is written as follows:

$$\begin{aligned} V_t^C &= \sum_{n\in\mathcal{N}} A_{C,t} \left(\prod_{a\in\mathcal{I}} K_{a,n,t}^{\theta_a} \right)^{\alpha} L_{n,t}^{1-\alpha} \\ &= \sum_{n\in\mathcal{N}} A_{C,t} k_t^{\alpha} L_{n,t} \\ &= A_{C,t} k_t^{\alpha} L_t. \end{aligned}$$

It follows immediately that the real GDP growth rate is

$$g_{V^C} = g_{A_C} + \alpha g_k + g_L,$$

and ALP growth rate is

$$g_{V^C/L} = \underbrace{g_{A_C}}_{\text{direct effect}} + \underbrace{\alpha g_k}_{\text{indirect effect}}.$$

Thus the definition of the real GDP does affect the direct effect, but not the indirect effect. This is intuitive since the indirect effect comes from capital deepening, which is determined independent of the choice of numeraire.